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HUMAN RESOURCES

SUPPLY RATE AND EQUILIBRIUM INVENTORY OF AIR FORCE ENLISTED PERSONNEL:

A SIMULTANEOUS MODEL OF THE ACCESSION AND RETENTION MARKETS INCORPORATING FORCE LEVEL CONSTRAINTS

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The objective of this research is to develop and test a general model of the Air Force manpower market. Previous studies of manpower supply have had mixed results, some even indicating that enlistments are negatively related to the Air Force wage. This study indicates that previous work failed to account for the simultaneous determination of enlistments and retentions and so has misinterpreted their regressions as supply equations when they are, in reality, reduced form equations resulting from the joint interaction of Air Force demand and enlistment supply activity.

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A stochastic process model is used to represent the Air Force manpower market. The variables which determine the levels of the steady state manpower inventory, personnel turnover and new enlistment flows are the cont'

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mandated force level requirement, Air Force and civilian wages, the unemployment rate and the perceived civilian returns to Air Force provided training. Because both the enlistment rate as well as the planned duration of service are treated in the model it is possible to trace the impact of changes in the determining variables upon retention and enlistments.

The empirical work rejects the standard regression "supply" approach and supports the simultaneous demand and supply approach developed in the study. The results indicate that: (a) changes in the force level account for the bulk of the changes in enlistment rates during the all volunteer force period, (b) higher Air Force wages have improved retention and reduced demand for new enlistments thereby leading to lower observed accession rates, (c) improvement in civilian job opportunities and wages reduce retention, increase enlistment demand and the new accessions show somewhat lower Air Force Qualifications Test scores, and (d) previous estimates of military manpower supply elasticities are shown to be biased downward from the true elasticity.

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I. INTRODUCTION AND SUMMARY

Research centering on estimating the supply of enlistees to the military began assuming increasing importance as early as a decade prior to the elimination of the draft. Amid the current suggestions that a return to some form of draft may be necessary, this research once again takes on an important role. The problem is basically one of raising the necessary manpower for the military at a direct cost that the American public is willing to bear. It is well known that the draft reduces the direct cost of raising the required manpower by forcing those conscripted to pay an indirect tax. The draft also raises other issues the most important of which is how to decide who is drafted.

Assuming that the all volunteer force (AVF) is to continue it is imperative that a complete understanding of the military manpower market be developed. The past work on this subject has concentrated on the career choice problem. That is, potential enlistees have been viewed as making their decisions based on a comparison of the present values of career earnings in the military versus civilian life.[1] Given tastes for the military it is argued that an individual opts for a military career when the present value of military earnings relative to the present value of civilian earnings is larger than some critical value. This critical value depends on the individual's tastes for the military.

This traditional theory results in prediction statements which can and have been tested.[2] In these tests, the rate of enlistment has been related to relative wages, the qualified military available (a population estimate), unemployment, and recruiting effort. The results obtained from the effort have been mixed but on the whole, at least from outward appearance, satisfactory for the period before the end of the draft. In a highly cited post AVF study, Cooper found what seem to be reasonable results using semi-annual data.[3]

This present study considers the market for Air Force enlisted personnel. The approach taken, though well grounded on economic principles, departs substantially from the existing literature on military manpower supply. This existing literature has attempted to estimate the relation between supply and its determinants – prime consideration has been given to the effect of wages and unemployment on the supply of enlistments through attempts to estimate supply elasticities. The problems with this literature are manifest to a careful reader; wage supply elasticities, as estimated, obey no orderly principles. The effect of the military wage on enlistments is as often negative as positive; it exhibits no stability over time, place, service or manpower pool, and is extremely sensitive to the form of the chosen estimating equation. It is literally possible, through judicious selection of the study cited, to find a magnitude and direction to support any point of view.

This unsatisfactory state is due to a well-known problem – the identification problem. Existing studies have failed to account for the fact that observed enlistments are jointly determined by demand and supply considerations and therefore have failed to identify the supply curve. What has been estimated is an improperly specified reduced form equation which represents the equilibrium of demand and supply. While some authors have realized they have an identification problem in their studies, it has never been clear how to specify the demand and supply sides of the military manpower market in order to resolve the problem.

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[1] See the excellent paper by Fisher (1969) for a discussion of this view.

[2] See Fisher (1969).

[3] See Cooper (1977).

Another important problem with the previous work results from the treatment of the military enlistment decision as a decision concerning career choice. In fact, only a small percentage of enlistees ever make the military a career or ever intend to do so, a fact well known to Air Force recruiters who emphasize the civilian value of Air Force training in their recruiting programs.

This study attempts to advance the analysis of these issues through formulation of a new model that links force level requirements, personnel inventories and the retention and accession markets in a simultaneous system. In addition, a new model of the enlistment decision which treats the duration of service as part of the enlistment decision process has been developed.

This report consists of six sections. Section I contains an introduction and a summary of the project. In Section II the theory of the enlistment decision is developed. In Section III the results of Section II are used to develop a general model of the equilibrium military force. In Section IV, the theoretical structure (developed in Section II and III) is related to the Air Force manpower experience. In Section V, the empirical evidence relating to previous work on the supply of enlistees is presented. Section VI contains the transition from theory to empirical work and reports the empirical work.

Summary

1. *The Air Force enlistment decision.* The traditional career choice theoretic approach to the Air Force enlistment decision fails to account for the fact that the mean length of service is from 4.5 to 5.2 years. That is, the mean of the time from enlistment to separation from the Air Force, for whatever reason, has ranged from 4.5 to 5.2 years for the 1969 to 1976 period. Thus, the median enlistee treats the Air Force not as a career but as a significant work experience in a working life cycle. A model of individual choice is formalized in this report which yields the optimal distribution of total working life between military and civilian alternatives. In this approach the typical individual has a planned working life that encompasses both military and civilian employment.

The theory has two important properties in explaining observed enlistment behavior. First, it makes the enlistment decision more than just a matter of comparing the present values of alternative career earnings. In fact, given that an individual has a positive desired level of military service his enlistment decision depends on the relation between his desired length of stay and the minimum enlistment period. Any individual with a desired length of military service greater than the minimum will supply his services. Those individuals with desired lengths of military service less than the minimum enlistment must make an all or nothing decision between no military service or the minimum enlistment period.

The second important property of the theory is its ability to explain the mean length of military service. This aspect of the theory is important because of the crucial role played by the mean service time in the determination of the demand for enlistees. In fact, it is shown that the mean length of service is positively related to the military wage and negatively related to civilian wages and the probability of finding civilian employment. The greater the mean length of service, the smaller the flow of new enlistments required to maintain any given force level.

2. *The Air Force manpower market.* The Air Force manpower inventory is jointly determined in two markets – the accession market and the retention market. A stochastic process model is developed which characterizes each of the markets. Viewing the allowable force as the number of servers in the process allows the mean length of service to be treated as the inverse of the service rate; i.e., one over the mean length of service is the expected number of times a particular position turns over in any given year. For the Air Force each position turns over approximately 0.2 times per year implying a five year expected length of service.

The mean length of service, as pointed out above, is determined by military and civilian wages. The allowable force level, on the other hand, is mandated by Congress. These two factors together determine the Air Force demand for enlistees. That is, the total number of positions times the number of times per year each position turns over yields the number of new enlistees that must be found if the force level is to be maintained.

On the supply side; i.e., the accession market, the rate at which new potential enlistees arrive is also a function of military and civilian wages. Since losses from the existing force do not occur at a constant rate, but rather are randomly distributed around a fixed mean, the demand for replacements is also random. In a similar fashion potential enlistees do not arrive in a constant stream, but such arrivals are also random.

As a result of this uncertainty on both sides of the market, not all potential enlistees will be able to enter the Air Force at the particular time of their choosing. In addition, the Air Force may not find enlistees always waiting to fill vacant slots. The implication follows that the actual force level is not a constant. Also, some potential enlistees are lost because they cannot enter the Air Force at their most preferred time; i.e., they balk by refusing to wait for induction.

These ideas have been utilized to derive the equilibrium level of accessions. This equilibrium level of accessions is not a supply curve of enlistees, but is rather an equation describing the equilibrium of supply and demand. In effect the Air Force trade off between wages, expected force level and quality can be developed. Importantly, the Air Force can choose only two of these three important variables and the market determines the third. The Air Force can fix wages and expected force level and then accept the resulting quality of personnel. On the other hand the Air Force can fix wages and quality and accept the resulting expected force level.

3. *The theory and actual Air Force manpower experience.* A great many studies have estimated the elasticity of supply of enlistees in order to estimate the costs of operating an all volunteer force. All of these studies have failed to recognize the joint determination of the demand for and supply of enlistees. In fact, once account is taken of this joint determination it is apparent that previous estimated equations are actually equations depicting the equilibrium of supply and demand forces. Accordingly the estimates of supply effects are not directly attributable to supply at all.

What is done here is to derive the expected signs of the coefficients of the equilibrium equations. For example, consider the effect of an increase in the military wage on the level of enlistments. First, such a change increases the mean length of stay in the military thereby reducing the demand for enlistees. Second, the change also increases the rate at which potential enlistees arrive. Thus, the Air Force can maintain its previous force level with a smaller number of enlistees or can have a larger force level with the same or a greater level of enlistments.

Depending on the change in the force level, the net impact of changes in the military wage on enlistments can be positive or negative. As it happens, the period covered by the majority of past studies was one in which this net effect was positive. The AVF period seems to be a period in which the net effect of military wage changes on new enlistments is negative.

4. *Some comments on past empirical work.* Some detailed comparisons between the theory developed in this report and the empirical work done by Fisher (1969) and Grissmer, Amey, Arms, Huck, Imperial, Koenig, Moore, Sica, and Szymanski (1974) was performed as a preliminary test of the usefulness of the approach. Fisher used data from 1957 to 1965, a period for which this theory would suggest a negative effect on accesssions for his wage variable (civilian wage divided by military wage). Fisher's equation was re-run for the 1969–1976 period. As suspected, for this latter period, the effect of Fisher's wage variable on accessions was significantly positive implying that decreases in Air Force wages increase supply.

The model developed in this report can explain this reversal in Fisher's results concerning the effect of the military wage on accessions. The actual level of accessions can be on the supply schedule or on the demand schedule depending on the level of military and civilian wages, force level requirements and the arrival rate of enlistees. Thus, the seeming paradoxically positive impact of rising civilian wages relative to military wages is explained by the fact that the force level was being reduced so that enlistment rates represented points on a shifting demand schedule.

5. *The empirical results.* In order to use the available data to estimate the model, the relation of the model to the data available must be derived. It is possible to relate the theoretical structure developed to its

empirical counterpart. In doing so the importance of the level of military wages relative to civilian wages for the level of net accession is demonstrated. Through some simplifying assumptions the entire planned working life earnings pattern can be collapsed into a function depending on only the beginning military and civilian wages. Thus, the supply of enlistees can be derived as a function of the military and civilian wage, the relevant population of entry-age individuals and the employment rate. In addition, the expected length of military service is a function of these same variables with the exception of the population variable.

Using the steady state results, derived in Section IV, the theoretical work can be transformed into its empirical counterpart. As a result of this transformation the relation between the estimated coefficients of the equations describing equilibrium and supply and demand schedules is apparent. In fact, it can be shown that the coefficient of, for example, the military wage in the usual supply equation is actually the result of two off-setting forces. One, the tendency of higher military wages to increase the mean force level due to increased supply. Two, the decline in the demand for accessions because of increased retention of personnel.

The actual estimates were performed on three measures of the accession rate each of which has a slightly different interpretation. First, accessions were measured as the rate at which individuals (whose paydate was the same as their service date) entered the Air Force. This measure of accessions is only a fraction of all those who enter the Air Force and is the closest thing we have to measuring the proportion of the time the system is not full, i.e., there is no wait for enlistment. Second, accessions were measured as the rate at which individuals signed up for the Air Force regardless of their service date. The second measure is the best estimate of the net supply of enlistees. Third, accessions were measured as the rate at which individuals entered the Air Force. This last measure is the best estimate of the Air Force replacement demand.

Because these three estimates are fundamentally different, we would expect the empirical results to differ and indeed this is the case. For example, the effect of changes in the wage ratio (Air Force wage divided by civilian wage) on accessions is consistently negative for the paydate = service date and service date regressions but positive for the paydate regressions. In general, however, the results are as expected and confirm the fact that the equations estimated represent equilibrium equations. Thus, the estimates cannot be used directly as supply or demand equations. The procedures developed do, however, provide a basis for attempts to estimate supply and demand, but a good deal more work is required to accomplish this.

Conclusions

This work has developed a significant class of models which provide a new way of looking at military manpower markets. Empirical estimates reveal that the observed accession rate can be explained by these models, but only by viewing enlistments as a result of interacting demand and supply forces.

This work has emphasized the importance of the demand side in the determination of the net rate of accessions. One major factor in the demand side is the force level. In fact, the empirical results indicate that changes in the force level are one of the primary sources of explanation for the Air Force manpower experience during the AVF period. The second major factor on the demand side is the retention rate which is related to the mean length of stay in the military. Our empirical results confirm the importance of this aspect of demand in that even after the effects of force level changes are removed the effect of an increase in the military wage on accesssions is negative.

It is clear from the empirical results and the consistency of the theory developed that the returns to additional work using this approach will be substantial. The preliminary empirical work makes it clear that a joint estimating technique must be used to estimate the retention and the accession markets. With the results of this joint estimation, reliable estimates of the supply and demand wage effects, without which a sensible Air Force manpower policy cannot be devised, should be available.

II. THE THEORY OF CAREER CHOICE AS APPLIED TO THE ENLISTMENT DECISION

In this section, the theoretical work is begun by modeling the problem of career choice. This is a necessary first step in developing the supply side of the Air Force enlisted personnel market. Moreover, this analysis is required because the existing theory of the civilian-military career choice is deficient in several respects and not adequate for purposes of the simultaneous consideration of the accession and retention markets. Of particular concern is the fact that the overwhelming bulk of Air Force enlistees do not spend their entire working life in the Air Force. If one looks at the distribution of departures from the Air Force by length of service, the mean of the length of service at time of separation in the Air Force has ranged between 4.5 to 5.2 years (see Appendix A), and less than 0.04 percent of individuals in the Air Force continue through the mandatory retirement age.[4] Thus, any analysis of the military-civilian choice must confront the fact that virtually all individuals who enlist will spend a substantial part, and in most cases the dominant part, of their working lives in the civilian sector.

Another issue that must be addressed (if a realistic analysis of the enlistment decision is to be obtained) is the fact that enlistment contracts are for a definite minimum tour of duty. Those individuals for whom this minimum is an effective constraint will be influenced by the level at which the specified minimum is set. For some, the minimum enlistment will induce them to enlist for longer than they otherwise would. For others, the minimum may be so high as to make them elect not to enlist. Lastly, the amount of training to be received and its eventual usefulness in the civilian sector enters into the enlistment decision. Clearly, training is relevant as an enlistment inducement only to the extent that the prospective trainee intends to apply that training later in civilian employment. Therefore, it is essential that the military enlistment decision be treated as constituting a choice of enlistment duration to be followed by a civilian career, rather than treating it as an all or nothing career choice. The relevant choice is the optimal distribution of the working years between both military and civilian employment, not between exclusive military or civilian careers.

The problem can be treated analytically by assuming that individuals make choices over goods, years in the military and years as a civilian. Assume that an individual's ranking over his choice space can be summarized in the following indirect utility function.

$$U = U(g, M, C) \tag{1}$$

where g is interpreted as the stream of goods available for consumption over the individual's lifetime, M is years of military service and C is years in civilian employment.[5] Expression (1) is a short-hand way of presenting the idea that individuals can rank various bundles of goods, years in the military and years as a civilian as to which bundle they prefer. In this decision it is assumed that consumption of goods represents the same level of consumption for every year of the individual's life. For convenience this level of consumption is labeled as *permanent consumption*. For any given level of permanent goods consumption the locus of combinations of military and civilian years of employment for which the individual is indifferent can be constructed. These loci, or indifference curves, are concave to the origin. In Figure 1, a set of such curves for various levels of permanent consumption is depicted. In the figure the horizontal axis

[4] Even those individuals who continue to the mandatory retirement age generally pursue civilian careers upon retirement.

[5] Herein goods consumption is treated as a permanent consumption stream. That is any planned variation in consumption expenditure is ignored. Such planned variation could be accounted for by expressing g as the present value of planned consumption expenditures. This approach does not affect the analysis to follow except to add to its complexity.

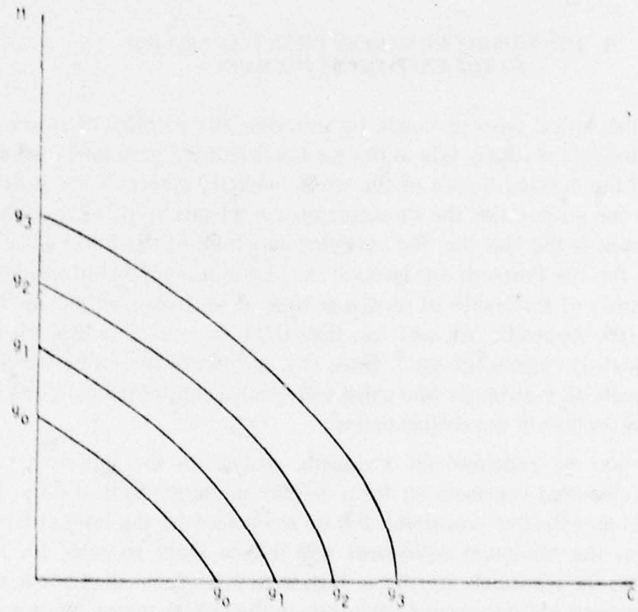

Figure 1. **Indifference curves between years in the military and as a civilian.**

measures years in the civilian labor force and the vertical axis measures years in the military. Each indifference curve represents, for a given level of consumption, various combinations of military and civilian employment which the individual finds are equivalent. Since total years of work is a "bad" rather than a "good," the greater the distance from the origin a given indifference curve lies, the greater must be the permanent consumption level since the greater is the total number of years of work.[6]

The concavity of the indifference curves implies that the individual believes that "variety is the spice of life." These curves imply that the individual is willing to devote more of his life working if he can divide his years between two occupations than if he specializes in either one. An individual, who has a distaste for variety, would have convex indifference curves for given levels of goods consumption. The intercept of the curves gives us an idea of the individual's relative feelings toward the military versus civilian life. The greater the civilian intercept is relative to the military intercept, the greater the preference for civilian life and vice versa.

Given the wage structure in the military and civilian occupations we can construct the relation between military years, civilian years and goods consumed. Let this relation be summarized by the implicit function

$$H(\overset{-}{g}, \overset{+}{M}, \overset{+}{C}) = 0 \tag{2}$$

where the signs of the partial derivatives of H are indicated above the appropriate argument. From (2) it is easily shown that increases in either military or civilian work years increase the level of goods consumption.

A given level of goods consumed per year will require a present value of earnings that will yield the revenue required for the goods consumption. Given the interest rate, the level of goods consumed will be proportional to the present value of the lifetime earnings stream. Thus, (2) can be written as

[6] This relation between utility and number of years worked follows because around the equilibrium value leisure must have positive marginal utility.

$$g = \gamma h(\overset{+}{M}, \overset{+}{C}) \tag{3}$$

where h(M, C) is present value of income and $\gamma > 0$ is a factor of proportionality.[7] The positive signs above M and C in (3) indicate the assumption that increases in either military or civilian working years increase goods consumption.

Now if civilian and military wages are fixed and equal, then h(M, C) will be linear in M and C. On the other hand, if training in the military raises civilian alternative wages then h(M, C) becomes strictly convex. The strict convexity of h(M, C) implies that the total labor input required for a given present value will be smaller than if an individual specializes in civilian employment.[8]

In Figure 2, an indifference curve for given permanent consumption g^o and an opportunity locus, which consists of the combinations of years of military and civilian work required to generate the present value necessary to purchase the consumption stream g^o have been superimposed. For purpose of illustration, it has been assumed that the present value function is linear in M and C. It will be useful to relax this assumption in the following analysis.[9]

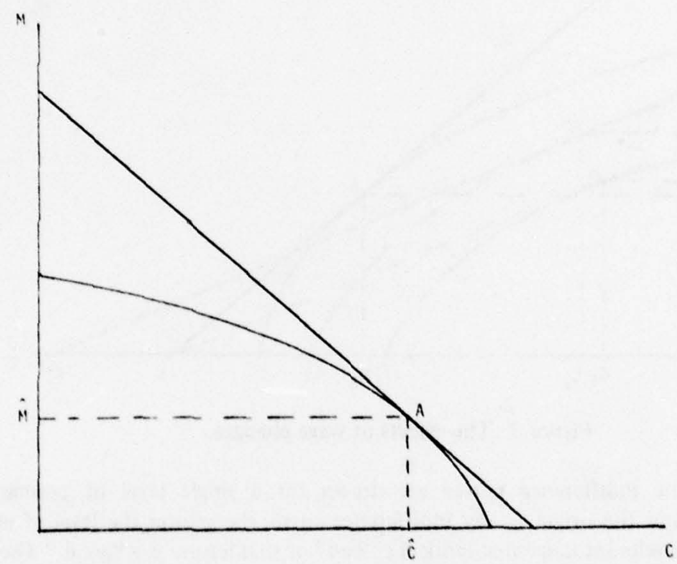

Figure 2. **The optimal military-civilian labor choice.**

The equilibrium in Figure 2 is at a point A with $\hat{M}$ years of military service and $\hat{C}$ years of civilian work. Note that the budget line (opportunity locus) represents the lowest number of working years necessary to allow consumption g^o. There are two important features to be noted. First, the number of years in the military depends on the relative wage in the military. In particular, the greater the military wage relative to civilian wages, the flatter the budget line and, accordingly, the greater the panned number

[7]In other words γ is a number such that a dollar amount equal to h(M, C) invested at the market rate of interest would just be exhausted at the individual's expected time of death.

[8]If the military wage is smaller than civilian wages then the constraint is concave and training has the effect of reducing concavity with the same result that if both M and C are positive, total labor input required for a given present value will be smaller than if the individual specializes in either M or C.

[9]The linearity assumption is not necessary for any of the results derived but simply makes the figures easier to follow. What is required is that the constraint be less concave than the indifference curve if an interior solution, i.e., a solution with positive values for both M and C, is to result.

of years of military service. Second, the individual's preferences for military versus civilian life are reflected in the slope of the indifference curve. The steeper the initial slope at the horizontal axis, the greater will be the number of military years chosen, if everything else remains the same.

Before proceeding further with this analysis an important issue should be addressed. This issue involves the effect of changes in the level of wages, which change the opportunity locus. These changes have two effects, an income and a substitution effect. Because the problem deals with a three-good world of permanent consumption, years in the military, and years as a civilian, the indifference map in Figure 1 represents a single level of utility. To clarify this point consider Figure 3.

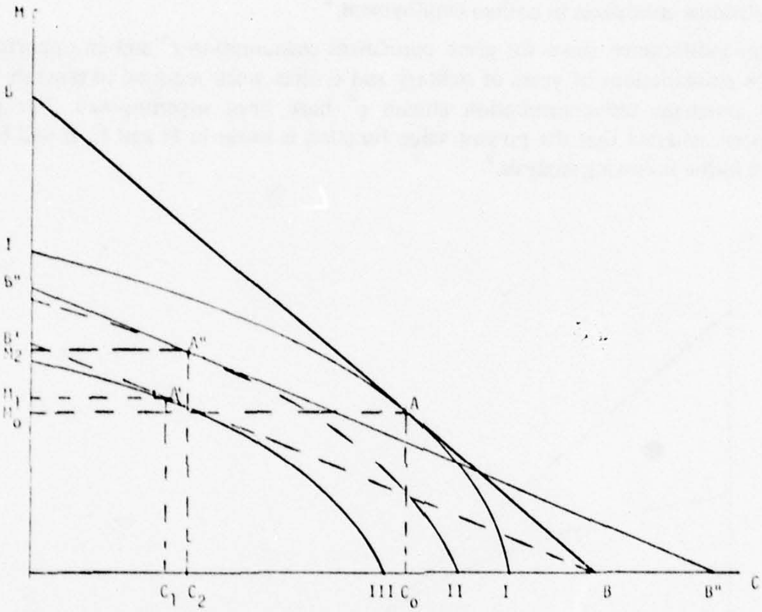

Figure 3. **The effects of wage changes.**

In Figure 3 the indifference curves are drawn for a single level of permanent consumption. Accordingly, the nearer the origin is any indifference curve, the greater the level of utility. That is, it is assumed that over the relevant range that work is a "bad" or that leisure is a "good." The opportunity locus in Figure 3, BB, is the combination of years in the military and years as a civilian that will yield the present value of income required to purchase the level of permanent consumption assumed in the construction of the indifference map.

Consider now an increase in the military wage. Such an increase results in the opportunity locus BB becoming flatter; that is, the number of years of military service required to yield the assumed permanent consumption stream has been reduced. This new opportunity locus is line BB′ in the figure. If the optimal level of permanent consumption remains unchanged, then the individual moves to point A′ and increases military service while reducing years as a civilian.

The implication of this equilibrium, however, is that the individual has consumed all of the change in his opportunity set as increased leisure. It should be expected that in reality the change in the opportunity set will be used up in increased goods consumption and leisure. Accordingly the relevant opportunity set must be consistent with a higher permanent consumption level. This new opportunity locus is denoted as line B″B″ in the Figure 3.

Since the original indifference map was constructed for a constant level of permanent consumption, a new indifference map must now be drawn. It is assumed that the dashed indifference curve is the relevant one for the new higher level of permanent consumption. Accordingly, the new equilibrium is at point A″ with greater years in both civilian and military than point A′. Point A″, however, still retains the substitution of military time for civilian time relative to the original equilibrium point A.

What has been developed above is a theory which determines the optimal mix of military and civilian life. Fundamentally this optimal combination is a function of the opportunity locus. The opportunity locus is itself a function of the wage structure in military versus civilian occupations. In fact, as shall be shown below, certain aspects of military training can result in an opportunity locus that is particularly conducive to the equilibrium choice containing a positive amount of military service.

The theory so far has concentrated on the desired length of stay in the military for those who desire a positive amount of military service. Equally important to the military are the determinants of the number of individuals who desire a positive level of military service. To discuss this question the previous discussion of the choice function (1) as it relates to individual tastes for military versus civilian life must be extended.

Figure 4 illustrates two sets of preferences for military versus civilian life. Both the indifference curves are concave and constructed for the same level of permanent consumption but there the similarity ends.[10] Indifference curve I I depicts an individual with high preference for the military as indicated by the fact that the number of years this individual will spend in the military, if he specializes, is approximately four times the specializing civilian years. Indifference curve II II depicts an individual with a relatively low preference for the military since it shows the opposite specializing preferences. Given equal wages in the two sectors, the opportunity locus will be a 45° line. The individual with high preference for the military will specialize in the military while the relatively low military preference individual specializes in civilian life.

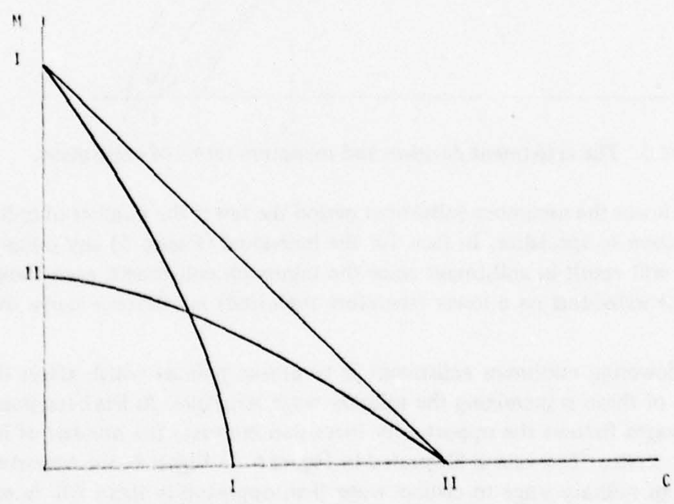

Figure 4. **Indifference curves with contrasting military propensity.**

Consider at this point the enlistment decision. First, the only candidates for enlistment are those with a positive level of desired military service. But, will all individuals with positive desires for the military

[10]This figure bears out a point made earlier. Namely, the concavity of the indifference curves does not determine the tastes for military vis-a-vis civilian life. Rather, the relative preference for these two occupations is represented by the ratio of the intercepts of the indifference curves.

enlist? Clearly not, since the military imposes certain minimum enlistment periods. Thus, if a minimum enlistment is four years, an individual with a desired military service of one year might not be expected to enlist although all those with desired military service of four or more years will enlist.

Figure 5 Illustrates this problem. The graph depicts an individual whose desired military service M_o is less than the minimum enlistment $\bar{M}$ (for pedagogical reasons $\bar{M}$ was made quite large). The individual must now make an all or nothing decision. He opts for some military service; i.e., moves to point B in Figure 5 and is on indifference curve 33, or specializes in civilian life and is on indifference curve 22. Since indifference curve 33 lies above indifference curve 22, it represents a lower level of utility and the optimal decision is to specialize in civilian life at point D.

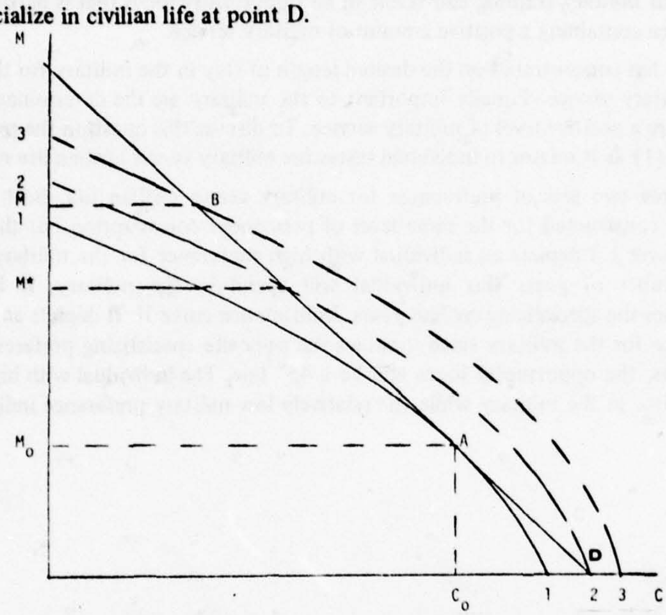

Figure 5. **The enlistment decision and minimum terms of enlistment.**

It is clear that the lower the minimum enlistment period the fewer the number of individuals that are forced to make the decision to specialize. In fact, for the individual (Figure 5) any minimum enlistment period smaller than M* will result in enlistment since the minimum enlistment, even though it is greater than M_o, will still put the individual on a lower (therefore preferred) indifference curve than indifference curve 22.[11]

An alternative to lowering minimum enlistment is to pursue policies which affect the opportunity locus. The most obvious of these is increasing the military wage structure. As has been pointed out above, an increase in military wages flattens the opportunity locus and increases the number of individuals with positive optimal military service. This case is illustrated in Figure 6. In Figure 6, the opportunity locus B′B′ represents a higher ratio of military wage to civilian wage than opportunity locus BB. In equilibrium with opportunity locus BB the individual specializes in civilian employment while with opportunity locus B′B′ he chooses a positive level of military service.

Alternatively, the military can offer training which can affect the individual's future civilian earnings. Such training will change the relative wage in favor of civilian occupations after it is received. Accordingly, this training makes the opportunity locus convex to the origin. Figure 7 illustrates the effect of the military training individuals in skills which have civilian value. Consider point A in Figure 7. This point is the

[11] Note that indifference curve 22 represents the indifference curve the individual will be on if he specializes in civilian employment.

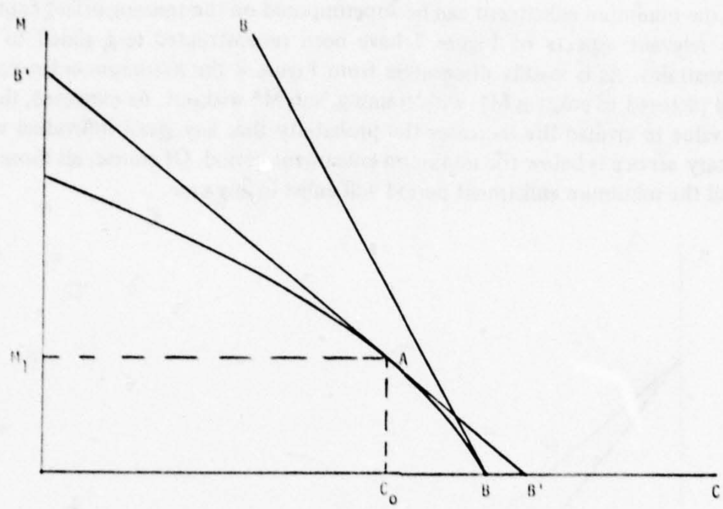

Figure 6. **The military wage and optimal military years.**

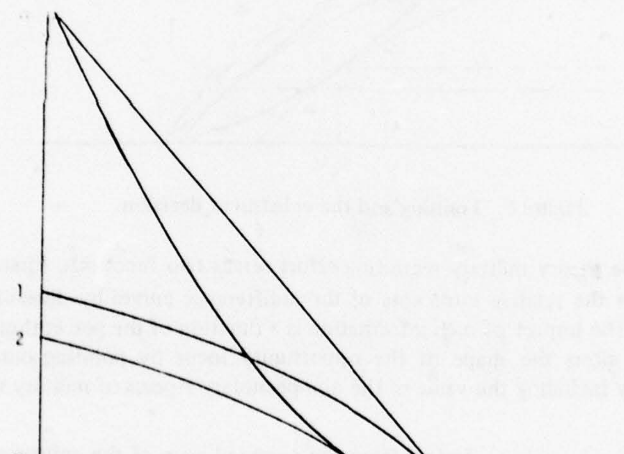

Figure 7. **Military training and optimal military years.**

optimal combination of military and civilian years, given the linear opportunity locus. Now introduce training which, after its completion, enhances civilian earnings. In effect this training reduces, for any given number of years of military service, the number of civilian years necessary to maintain the desired permanent consumption level. Accordingly, the training opportunity locus is represented by the convex opportunity locus and the new equilibrium is at point B on indifference curve 22. Now the individual optimizes by choosing a positive level of military service.[12]

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[12]The effect of military training on the constraint is discussed further in Section VI of this report.

The impact of the minimum enlistment can be superimposed on the training effect captured in Figure 7. In Figure 8, the relevant aspects of Figure 7 have been reconstructed and added to the minimum enlistment period constraint. As is readily discernible from Figure 8 the minimum enlistment required to induce the individual pictured to enlist is M_T^* with training, and M^* without. As expected, the introduction of training that has value in civilian life increases the probabilty that any given individual will enlist if his desired level of military service is below the minimum enlistment period. Of course, all those whose desired service periods exceed the minimum enlistment period will enlist in any case.

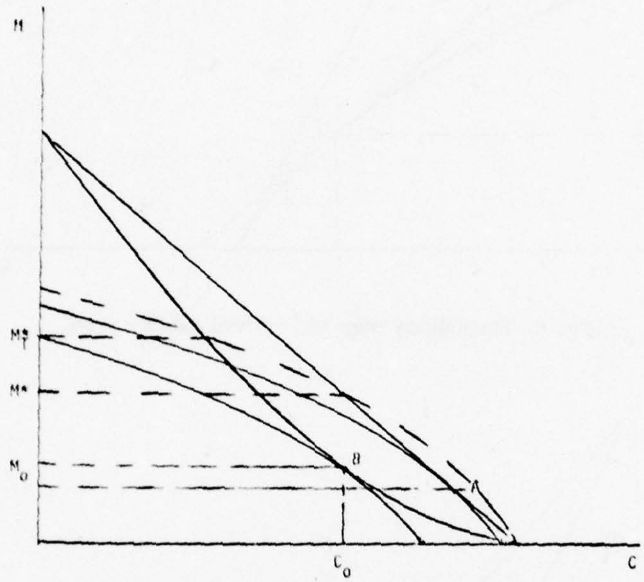

Figure 8. **Training and the enlistment decision.**

In view of the above theory military recruiting effort serves two functions. First, it alters tastes for the military; i.e., it alters the relative intercepts of the indifference curves by disseminating information concerning the military. The impact of such information is a function of the perception individuals have of the military. Second, it alters the shape of the opportunity locus by pointing out to individuals the advantages of the military including the value of the non-pecuniary aspects of military life and the value of the training received.

What has been done above is to deviate from the accepted view of the enlistment decision making process. Past studies have concentrated on the career choice aspects of the enlistment decision. This approach, while yielding fruitful implications, does not account for the fact that for almost all its members the military is not a lifetime occupation. The approach developed here has been to view the relevant choice set as the distribution of the expected working life over military and civilian years. This allows the simultaneous discussion of the enlistment and length of service decisions.

The ultimate number of individuals who present themselves for enlistment, the supply of enlistees, and the expected length of military service depend on the distribution of individuals by tastes and opportunities. As the theory has shown, the greater the military wage relative to the civilian wage the more likely is any individual to have a non-zero desired length of military service. Thus, the supply of enlistees will be positively related to relative wages. Increases in the military relative to civilian wage also increase the desired length of stay in the military. The importance of this latter result will be shown in the subsequent sections of this report. Suffice it to say here that the increased desired length of stay decreases turnover and thereby reduces demand for enlistees.

Mathematical Appendix to Section II

Consider the utility function

$$U = U(g, M, C) \tag{4}$$

and the opportunity set

$$g = H(M, C) \tag{5}$$

The maximization of (4) subject to (5) requires that

$$U_g = \lambda \tag{6}$$

$$U_m = -\lambda H_m \tag{7}$$

$$U_c = -\lambda H_c \tag{8}$$

where λ is the Lagrangian multiplier. In this case λ has the interpretation of being the marginal value of permanent consumption. Thus, (7) and (8) can be interpreted as requiring that the marginal disutility of military or civilian time equal the marginal gain in utility from the additional permanent consumption available because of the additional military or civilian time. In addition it is required that the matrix of second derivatives of the augmented function bordered by the first derivatives of the constraint (the bordered Hessian) be negative definite

$$D = \begin{bmatrix} U_{gg} & U_{gm} & U_{gc} & -1 \\ U_{mg} & U_{mm} + \lambda H_{mm} & U_{mc} + \lambda H_{mc} & H_m \\ U_{cg} & U_{cm} + \lambda H_{cm} & U_{cc} + \lambda H_{cc} & H_c \\ -1 & H_m & H_c & 0 \end{bmatrix} \tag{9}$$

where U_{ij} and H_{ij} are respectively the second partial derivatives of the utility function and opportunity locus.[13]

From the system (5) through (8) the equilibrium values for permanent consumption, desired years of military service and years in civilian life as function of the parameters of the system can be derived. Assuming that the function H is simply the present value of the military and civilian income stream, (5) can b written as

$$H(M, C) = \int_{o}^{M} e^{-rt} W_m(t;\theta) dt + \int_{M}^{M+C} e^{-rt} W_c(t;m,\phi) dt \tag{10}$$

where r is the rate of interest and θ and ϕ are shift parameters to be used to reflect wage changes. If it is assumed that θ and ϕ are proportional to the initial military and civilian wages then g, M and C can be written as

$$g = g(W_m, W_c) \tag{11}$$

[13]The reason for the second derivatives of the constraint appearing in the Hessian proper is because the constraint is non-linear.

$$M \quad = M(W_m, W_c) \tag{12}$$

$$C \quad = C(W_m, W_c) \tag{13}$$

The derivatives of (11) through (13) can be obtained by totally differentiating the first order conditions and the constraint and solving for the derivatives. These derivatives are

$$\frac{\partial X_i}{\partial W_j} = -\lambda H_{iWj} \frac{D_{ij}}{|D|} + H_{Wj} \frac{D_{oi}}{|D|} \tag{14}$$

where i = g, M, C ; j = M, C; D_{ij} is the co-factor of (9) associated with the ith row and jth column and D_{oi} is the co-factor associated with the border row and the jth column. In effect the derivatives can be separated into two effects, a substitution effect and an income effect.

The substitution effect consists of the term $[-\lambda H_{ij}\ D_{ij}/|D|]$ and for i = j must be positive since $H_{iw_j} > 0$, $\lambda > 0$ and $(D_{ij}/|D|) < 0$ by the negative definiteness of D. The income effect consists of the term $(H_{w_j} D_{oi}/|D|)$ which is negative for i,j = M,D and positive for i = g and j = M,C. The signs of the income terms require the assumption that permanent consumption and less years working in either occupation are normal goods. That is, it is assumed that as individuals become wealthier they consume more and work less if their alternative costs remain unchanged.

To illustrate, Figure 9 shows a graphical representation of the income and substitution effects. In the figure the initial equilibrium is point A on constraint $b_o b_o$. Let the military wage rise so that the new constraint is $b_o b_2$. Now the substitution effect is the movement from point A to point A′ on the original indifference curve.

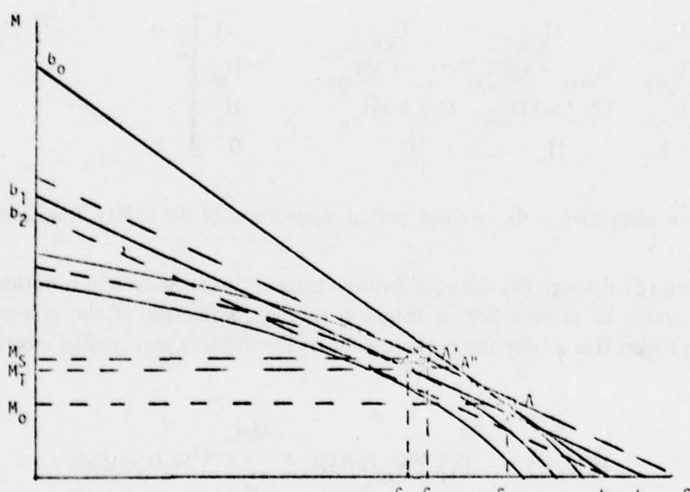

Figure 9. **Income and substitution effects.**

The constraint at this point will purchase a higher level of permanent consumption than originally consumed. This higher level of consumption represents an increase in income and will be consumed partly as an increase in leisure; i.e., a shortening in working life, so that both military and civilian years decline. This decline is shown as the movement from point A′ to A″. The indifference curve associated with point A″ has greater permanent consumption and lower working years than the original equilibrium position. This higher consumption follows because the constraint on which point A″ lies (b_1 b_1) intersects the C axis to the right of b_o which, with constant civilian wages, implies greater present value of earnings.

Now if it is assumed that substitution effects dominate income effects, then the derivatives are

$$\frac{\partial g}{\partial W_m} > 0 \; ; \; \frac{\partial g}{\partial W_c} > 0$$

$$\frac{\partial M}{\partial W_m} > 0 \; ; \; \frac{\partial M}{\partial W_c} < 0 \tag{15}$$

$$\frac{\partial C}{\partial W_m} < 0 \; ; \; \frac{\partial C}{\partial W_c} > 0$$

The distinction between the approach developed here and the usual career choice theory is now apparent. In the life cycle approach an increase in military wages increases desired time spent in the military vis-a-vis civilian employment. Additionally, an increase in the military wage increases the proportion of all individuals who have non-zero desired military time and therefore are possible enlistees. Finally, an increase in the military wage increases the probability that an individual with a positive level of desired military time will enlist for any given minimum enlistment period.

In contrast, the usual way of viewing enlistment is as a career choice. In this approach the present value of military wages is compared to the present value of civilian wages. Once the military present value relative to the civilian present value exceeds some critical number, the individual enlists; i.e., chooses a military career. The career choice approach does not explain why the mean length of stay in the military is a small portion of the expected working life and accordingly the approach is in need of amending.

III. A MODEL OF THE AIR FORCE ENLISTED PERSONNEL ACCESSION AND RETENTION MARKETS

The Air Force inventory of personnel is determined through demand and supply forces in two inter-connected markets – the accession market and the retention market. The overall demand for personnel is determined by force level requirements. This is a stock demand; i.e., the demand that a certain personnel inventory be maintained. Losses from the inventory occur randomly but at a mean rate which depends on the decisions made by individuals completing terms of enlistments. These individuals may retire (if they qualify), extend, reenlist or opt out of military service and their decisions to do so determine the supply of experienced personnel. It will be convenient to call the market where this supply is relevant the continuation or retention market. The demand for continuations is treated as primarily dependent on the force level requirement and the desired experience mix of the Air Force enlisted personnel. A high force level requirement or a demand for high levels of experience would result in high continuation demand.

The other market – the accession market – is linked to the continuation market through the demand side. New accessions are demanded to replace losses from the personnel inventory. Since these losses are random, the demand for replacement will also be random. In elemental terms think of the system as a one-for-one random inventory model. Each time a loss occurs a demand for a new accession is created, thus losses from the inventory are replenished on a one-for-one basis and the *demand for accessions* is related to the *supply of continuations*. The supply of accesssions is treated in a fairly conventional manner here and depends on relative military and civilian earnings prospects. The only departure from the standard accession supply analysis is to let the accession rate be a random variable and to allow some proportion of willing enlistees to be discouraged, and hence withdraw, if the wait to be inducted is too long.

The definition of an equilibrium in the model differs somewhat from the usual one which says that the quantities demanded and supplied are equal at the equilibrium wage. In the present context this need not be the case. A steady state equilibrium exists if the personnel inventory stabilizes or has an expected value greater than zero and less than infinity. That is, the force level, while it is a random variable, must

have an average and can not fall to zero or become infinite through time. If the force level shows no systematic tendency to shrink or grow without limit and fluctuates around some average level, then the system is in equilibrium. There is no need for the quantity demanded to equal the quantity supplied in the accession or retention markets to attain equilibrium. In fact, it will be argued that their equality implies that the Air Force, in that situation, cannot invoke any quality standards such as AFQT score minimums. Under the appropriate definition of equilibrium there may be an excess of applicants in one or both markets, with quality rationing making the quantity taken in either market equal to the quantity demanded so long as there is an excess supply. Provided only that a steady state exists, the markets are in equilibrium.

There are a variety of equilibria which are determined by the wage structure. A high turnover-low mean experience regime would be achievable through a system of wages that did not provide a large premium for experience. Alternatively, the same mean inventory could be achieved with low turnover and a high proportion of experienced personnel by paying a greater premium in the retention market. The optimal experience mix would depend upon the productivity of experience and its costs.

There are two essential components of the Air Force manpower market, the rate of arrivals of potential enlistees and the rate of withdrawal of existing Air Force manpower. For any given Air Force manpower level the number of arrivals who enlist in the Air Force must exactly equal the number of withdrawals from the existing force. The Air Force controls to some degree both of these groups since it can, by altering its minimum standards for both enlistees and re-enlistees, affect the numbers of searchers who stay with the Air Force and the number of existing Air Force personnel who leave.

The Supply Process

Consider a world in which individuals interested in the possibility of an Air Force career search at Air Force recruiting stations. Let the time that elapses between the arrival of successive searchers be exponentially distributed; i.e.,

$$t \sim \alpha\, e^{-\alpha t} \tag{16}$$

with expected value equal to $(1/\alpha)$. Thus, the probability that the time between arrivals T is greater than t is

$$P(T \geqslant t) = e^{-\alpha t} \tag{17}$$

which is a declining function of t. From the assumption that the time between arrivals is exponentially distributed it follows that the number of arrivals in any time interval of length t will be distributed Poisson so that,

$$n \sim \frac{(\alpha t)^n}{n!}\, e^{-\alpha t} \tag{18}$$

with expected value equal to αt. Essentially, it is being assumed that the arrivals come from a time-independent stochastic process with an expected number of arrivals per period of α so that in t periods one expects αt arrivals, or in an interval of unit length the expected number of arrivals is α. In the queueing literature α is referred to as the arrival rate.

Let α be a function of the Air Force wage, the civilian wage, the probability of acceptance in the civilian market and the quality requirement of the Air Force; e.g., the minimum AFQT and schooling standards. Thus, we write α as

$$\alpha = \alpha(\overset{+}{W_A}, \overset{-}{W_C}, \overset{-}{P_C}, \overset{-}{Q}) \tag{19}$$

where the signs above the arguments of (19) represent the assumed signs of the derivatives of (19) with respect to that argument.[14] The probability of a searcher's acceptance by the Air Force depends on the standards set by the Air Force and the rate at which the Air Force inducts candidates.

On the demand side the Air Force has an allowable force level mandated by Congress with the advice of the Air Force. While this allowable force level is not necessarily that level most desired by the Air Force, it is true that the Air Force does influence Congress in its choices of the force level. Given the allowable force level, the rate at which the Air Force can induct enlistees depends on the rate of quits and fires. Since the Air Force can influence both quits and fires, it follows that the equilibrium in the retention market and the accession market are simultaneously determined.

The rate at which Air Force personnel leave the Air Force can be modeled by considering each position in the force to be akin to a server in a service process. In this manner the Air Force consists of F servers where F is the allowable force level. Let the expected stay in the Air Force be $(1/\mu)$ and assume that each fire or quit (where quits include all reasons for leaving the Air Force other than fires) is independent and that the time that any individual spends in the Air Force (S = service time) is exponentially distributed so that

$$S \sim \mu\alpha^{-\mu t} \tag{20}$$

Now assume that the allowable force is of size F and consists of F independently distributed individuals whose service time is distributed exponentially as in (20). In this case, the number of quits and fires (m) during any interval of length t will be distributed Poisson.

$$m \sim \frac{(F\mu t)^m}{m!} e^{-F\mu t} \tag{21}$$

The expected number of departures from the Air Force during an interval of unit length is from (21) equal to $F\mu$. In the queueing literature $F\mu$ is referred to as the service rate.

The rate at which a given position in the Air Force turns over during a unit interval is equal to μ. The inverse of this rate, as pointed out above, is the expected time of service before departure from the Air Force. For example, if the unit interval is a month then μ is a number smaller than unity and accordingly $(1/\mu)$ is greater than one. In fact evidence suggests that for 1976 μ was approximately 0.209 implying that the mean service time was 4.78 years.

The rate at which positions turn over in the Air Force is a function of civilian versus military opportunities. Accordingly, this rate will be a function of the civilian wage and the probability of finding civilian employment on the one hand and the military wage and quality requirements on the other. In particular, increases in the civilian wage and/or the probability of finding civilian employment will increase the turnover rate and increases in the military wage and/or decreases in the Air Force quality requirements will decrease the turnover rate. Thus, μ can be written

$$\mu = \mu(\overset{-}{W_A}, \overset{+}{W_C}, \overset{+}{P_C}, \overset{+}{Q}) \tag{22}$$

where the signs above the arguments of (22) indicate the sign of the derivative of (22) with respect to that argument.[15]

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[14]Thus, the expected rate at which potential enlistees arrive is treated as being determined by those factors which in Section II were determined relevant for the enlistment decision.

[15]Thus, the expected stay in the Air Force is a function of those variables which were developed in the theory of the military/civilian choice problem discussed in Section II above.

The Air Force Market

Given that arrivals to and departures from the Air Force are random there will be times that, for any given level of manpower quality (Armed Forces Qualification Test/AFQT score or other qualifications), the Air Force will not be able to accept all arrivals and other times that arrivals will not be sufficient to maintain the force level. In effect, a queue of potential inductees will develop at some times and at others a queue of Air Force vacant positions will occur.

The willingness of arrivals to tolerate queues will differ but on the average there will be a positive probability that an individual will balk, i.e., refuse to wait in a queue of a given size. This probability will depend on the expected length of the wait in the queue, the civilian wage, the military wage, and the probability of obtaining civilian employment. The expected wait in the queue depends on the length of the queue and the rate at which Air Force positions become available; i.e., the service rate. The expected wait ($\bar{\omega}$) given that there are n in the queue is

$$(\bar{\omega} | n) = \frac{n}{F\mu} \tag{23}$$

Let ω be the length of time prospective recruits are willing to wait in the queue, than ω can be written

$$\omega = \omega(\overset{+}{W}_A, \overset{-}{W}_C, \overset{-}{P}_C, \overset{-}{Q}) \tag{24}$$

so that the higher the Air Force wage the longer prospective recruits will wait and the higher the civilian wage or the probability of obtaining civilian employment the less time prospective recruits are willing to wait in the queue. An increase in the Air Force quality requirements affects the time prospective recruits will wait in the queue because it restricts the queue members to those with better civilian opportunities thereby increasing the alternative cost of waiting.[16]

Given (23) and (24) the maximum queue length B can be written as

$$B = \omega F\mu = B(W_A, W_C, P_C, Q, F) \tag{25}$$

The derivatives of (25) depend, except for the force level, on the difference between the elasticity of the waiting time and service time functions. In particular

$$\frac{\partial B}{\partial W_A} = \frac{\omega\mu F}{W_A}(\epsilon_{\mu W_A} + \epsilon_{\omega W_A}) \tag{26}$$

$$\frac{\partial B}{\partial W_C} = \frac{\omega\mu F}{W_C}(\epsilon_{\mu W_C} + \epsilon_{\omega W_C}) \tag{27}$$

$$\frac{\partial B}{\partial P_C} = \frac{\omega\mu F}{P_C}(\epsilon_{\mu P_C} + \epsilon_{\omega P_C}) \tag{28}$$

$$\frac{\partial B}{\partial Q} = \frac{\omega\mu F}{Q}(\epsilon_{\mu Q} + \epsilon_{\omega Q}) \tag{29}$$

[16] The cost of waiting is the earnings lost because permanent employment opportunities must be foregone. Hence, the higher the AFQT requirement the greater the cost and accordingly the shorter the tolerable wait.

$$\frac{\partial B}{\partial F} = \omega\mu > 0 \tag{30}$$

From (22) and (24) it follows that $\epsilon_{\mu i}$ and $\epsilon_{\omega i}$ are opposite in sign for all i. Essentially the direction of the effect of a parameter change (say an increase in the Air Force wage) on the balking queue length depends on how much the increase in Air Force wage increases the time potential enlistees are willing to spend in the queue versus how much this same change reduces the speed at which the queue moves.

The solution for the expected force level and the expected queue length can be derived using the fact that a Markovian birth-death process has been assumed.[17] Given that α is the rate at which prospective enlistees arrive at the Air Force and that the mean length of stay in the Air Force is $(1/\mu)$ the probability that the Air Force has exactly n enlisted personnel is

$$P_n = \frac{F^B F! \mu^{K-n}}{n! \alpha^{K-n}} p_K \qquad F > n \geqslant 0 \tag{31}$$

where K is the maximum system size; i.e., the sum of the allowable force F plus the maximum queue length B. In addition, the probability that the Air Force is at maximum allowable strength and that a queue of potential enlistees exists; i.e., the probability that the total system size (queue plus Air Force personnel) is exactly n is

$$p_n = \left(\frac{F\mu}{\alpha}\right)^{K-n} p_K \qquad K > n \geqslant F \tag{32}$$

Note that the probabilities in (31) and (32) all depend on the probability that the system is *full*. In fact p_K must be positive for a system steady state to exist. That is, there must exist a positive probability that the Air Force would be at full strength and that the queue of potential enlistees is at the balking queue length. The above dependence on p_K could be translated into a dependence on any arbitrary system size so that p_K would then represent the largest system size having a positive probability.[18]

Since the set $\{p_i: i = 0, \ldots K\}$ represents a probability distribution, $\sum_{i=0}^{K} p_i = 1$. Thus from (31) and (32) it follows that the probability that the system is full is

$$p_K = \left\{ F^B F! \sum_{n=0}^{F-1} \frac{1}{n!} (\mu/\alpha)^{K-n} + \sum_{n=F}^{K} (F\mu/\alpha)^{K-n} \right\}^{-1} \tag{33}$$

From (33) it can be seen that p_K is a function of the allowable force level, the arrival rate, the mean service time and the balking queue. Thus, from (19), (22) and (23) it follows that p_K can be written as

$$p_K = p(\overset{+}{W_A}, \overset{-}{W_C}, \overset{-}{P_C}, \overset{-}{Q}, \overset{-}{F}) \tag{34}$$

where the expected signs of the derivatives of (34) have been written above the appropriate argument.

From the earlier discussion of balking it is apparent that the probability that the system is full is exactly the probability that the next potential recruit arriving will balk. That is, p_K represents the

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[17] A Markovian birth-death process is one in which at most one entry (birth) or departure (death) occurs at any point in time. In addition, successive births and deaths are independent.

[18] If balking does not occur then there is no finite maximum size to the system. In this case the normalizing probability is usually the probability of the system being empty.

proportion of the time that the system will be full so that if potential enlistees arrive at random intervals the probability that any arrival will balk will be equal to the proportion of time that the system is full; i.e., p_K. An increase in the Air Force wage affects the full system probability in several ways. First, increased Air Force wages increase the expected length of stay in the Air Force and thereby reduce μ. Second, increased Air Force wages increase the desirability of the Air Force and thereby increase arrivals, α. Third, if the Air Force is more desirable the arriving potential enlistees are willing to wait longer, but as has been shown the effect of this increase in the maximum wait on the balking queue is indeterminant because of the reduction in μ.

If it is assumed that the effect of changes in the Air Force wage, the civilian wage, the probability of civilian employment and quality on the balking queue are small because of their offsetting effects on willingness to wait (ω) and speed of the queue (μ), then the derivatives of (34) can be solved for unambigously. In this case the derivatives are

$$\frac{\partial p_K}{\partial W_A} = \Theta\left(\frac{\mu_A}{\mu} - \frac{\alpha_A}{\alpha}\right) > 0 \tag{35}$$

$$\frac{\partial p_K}{\partial W_C} = \Theta\left(\frac{\mu_C}{\mu} - \frac{\alpha_C}{\alpha}\right) < 0 \tag{36}$$

$$\frac{\partial p_K}{\partial P_C} = \Theta\left(\frac{\mu_P}{\mu} - \frac{\alpha_P}{\alpha}\right) < 0 \tag{37}$$

$$\frac{\partial p_K}{\partial Q} = \Theta\left(\frac{\mu_Q}{\mu} - \frac{\alpha_Q}{\alpha}\right) < 0 \tag{38}$$

where

$$\Theta = -p_K^2 \left[F^B F! \sum_{n=0}^{F-1} \frac{(K-n)}{n!} (\mu/\alpha)^{K-n} + \sum_{n=F}^{K} (k-n)(F\mu/\alpha)^{K-n}\right]$$

The bracketed term in Θ is exactly equal to $[(K-\bar{S})]$ where $\bar{S}$ is the expected system size which is necessarily less than the maximum system size K. Therefore, Θ is less than zero. But from (19) and (22) it follows that the difference in (35) through (38) is negative for the military wage and positive for all the other parameters. Since Θ is negative, the signs of (35) through (38) are as indicated.

The effect of changes in the allowable force size on the probability of the system being full is the result of two effects. First, an increase in the allowable force increases the service rate. Accordingly, the system will be full less often since it can work off queues more rapidly. Second, the increase in the service rate results in an increase in the queue length that will be tolerated. Because of the nature of the function describing p_K, the actual derivative of p_K with respect to F will not be evaluated. The derivative itself is

$$\frac{\partial p_K}{\partial F} = -\frac{B}{F}[1 + \ln(F\mu/\alpha)]p_K - p_K^2 (F\mu/\alpha)^B . \tag{39}$$

$$\left\{[\Gamma'(F+1) + F!\ln(\mu/\alpha)] \sum_{n=0}^{F-1} \frac{1}{n!} (\mu/\alpha)^{F-m} + \frac{1}{F} \sum_{n=F}^{K} (F-m)(F\mu/\alpha)^{F-m} + \ln(F\mu/\alpha) \sum_{n=F}^{K} (F\mu/\alpha)^{F-m}\right\}$$

where $\Gamma'(F + 1)$ is the derivative of the Gamma function. The sign of (39) cannot be evaluated analytically but it is reasonable to assume that (39) is negative.[19]

The expected force level can be derived using (31) and (32) and is

$$f = [F^B F! \sum_{n=0}^{F-1} \frac{n}{n!}(\mu/\alpha)^{K-n} + F \sum_{n=F}^{K} (F\mu/\alpha)^{K-n}]\, p_K \tag{40}$$

which can be written as

$$f = (\mu/\alpha)\{[F^B F! \sum_{n=0}^{F-1} \frac{1}{n!}(\mu/\alpha)^{K-n} + \sum_{n=F}^{K-1} (F\mu/\alpha)^{K-n}]\, p_K\} \tag{41}$$

Now by adding and subtracting p_K, the term inside the braces of (41) becomes

$$f = [(\alpha/F\mu)(1 - p_K)]\,F \tag{42}$$

Thus, the expected force level is proportional to the allowable force level with the factor of proportionality depending on the arrival rate of recruits, the turnover rate; i.e., the service rate, and the probability that the system is full; i.e., the balking probability.

The derivatives of the expected force level are

$$\frac{\partial f}{\partial W_A} = \phi\left(\frac{\alpha_A}{\alpha} - \frac{\mu_A}{\mu}\right) > 0 \tag{43}$$

$$\frac{\partial f}{\partial W_C} = \phi\left(\frac{\alpha_C}{\alpha} - \frac{\mu_C}{\mu}\right) < 0 \tag{44}$$

$$\frac{\partial f}{\partial P_C} = \phi\left(\frac{\alpha_P}{\alpha} - \frac{\mu_P}{\mu}\right) < 0 \tag{45}$$

$$\frac{\partial f}{\partial Q} = \phi\left(\frac{\alpha_Q}{\alpha} - \frac{\mu_Q}{\mu}\right) < 0 \tag{45}$$

$$\frac{\partial f}{\partial F} = -\frac{\partial p_K}{\partial F} > 0 \tag{47}$$

where $\phi = (\mu/\alpha)[(1 - p_K) - p_K^2 (K - \bar{S})]$. The bracketed term in ϕ consists of terms that are opposite in sign; however, the values of p_K and $\bar{S}$ are not independent. In fact, the larger is p_K, the greater $\bar{S}$. That is, the greater the probability that the system is full, the less the difference between the maximum system size and the expected system size. Assume that ϕ is positive since stability of the decision process used later requires this assumption.[20] With this assumption the expected force level is positively related to the Air Force wage and the allowable force level and negatively related to the civilian wage, the probability of obtaining civilian employment and the quality required by the Air Force.

[19] The evaluation of (39) requires numerical analysis. This analysis was performed for a limited range of the values of B, F, μ, and α and confirmed the negative sign of (39). A complete evaluation, however, has not been performed.

[20] Numerical analysis on a limited range of B, F, μ, and α confirms the assumption that ϕ is positive. A complete analysis has not been performed.

The expected number in the queue is from (32)

$$L = p_K \sum_{n=0}^{B} (B-n)(F\mu/\alpha)^n \tag{48}$$

where L is the number in the queue. After some algebraic manipulation (48) can be written as

$$L = \frac{p_K}{[1-(F\mu/\alpha)]^2} \left\{ (1+B)[1-(F\mu/\alpha)] - [1-(F\mu/\alpha)^{B+1}] \right\} \tag{49}$$

so long as $(F\mu/\alpha) \neq 1$. The expected queue length is always positive since for $(F\mu/\alpha) > 1$, the term in braces is a monotone increasing function of $(F\mu/\alpha)$ with a minimum of 0 at $(F\mu/\alpha) = 1$, and for $(F\mu/\alpha) < 1$ the term in braces is montone decreasing with a minimum of 0 at $(F\mu/\alpha) = 1$. The derivatives of L are as follows

$$\frac{\partial L}{\partial W_A} = \Psi\left(\frac{\alpha_A}{\alpha} - \frac{\mu_A}{\mu}\right) > 0 \tag{50}$$

$$\frac{\partial L}{\partial W_C} = \Psi\left(\frac{\alpha_C}{\alpha} - \frac{\mu_C}{\mu}\right) < 0 \tag{51}$$

$$\frac{\partial L}{\partial P_C} = \Psi\left(\frac{\alpha_P}{\alpha} - \frac{\mu_P}{\mu}\right) < 0 \tag{52}$$

$$\frac{\partial L}{\partial Q} = \Psi\left(\frac{\alpha_Q}{\alpha} - \frac{\mu_Q}{\mu}\right) < 0 \tag{53}$$

$$\frac{\partial L}{\partial F} = \frac{P_K}{(1-\delta)^3 F} [(B+1)(2-\delta)\delta^{B+2} + B(1-\delta)(1+\delta^{B+1}\ln\delta) - \delta(1+\delta)] + \frac{L}{P_K}\frac{\partial P_K}{\partial F} < 0 \tag{54}$$

where $\delta = [F\mu/\alpha]$ and $\Psi = \frac{P_K}{(1-\delta)^3} [(1-\delta^{B+1}) - (B+1)^B(1-\delta)] - \frac{1}{(1-\delta)} \left\{ \delta L[1+(K-\bar{S})] + L(K-\bar{S}) \right\}$

The evaluation of expected length of queue derivatives requires the sign of Ψ. For $\delta > 1$, the normal case, the first term on the right-hand side of Ψ is negative and small while the second term is positive.[21] Given Ψ positive the derivative of the expected queue with respect to the Air Force wage is positive and with respect to the civilian wage, the probability of finding civilian employment and Air Force quality requirements is negative. The derivative of the expected queue with respect to the allowable force is the sum of two effects both of which are the same sign. Thus, an increase in allowable force level reduces the expected queue.

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[21] For $\delta < 1$, $(1-\delta) > 0$ and the first term in ψ becomes positive and the second term negative. Numerical analysis on a limited range of values of B, F, μ, and α shows ψ to be positive.

Equilibrium in the Air Force Manpower Market

It follows from the foregoing analysis that of the three variables (Air Force wage, expected force level and expected quality), the Air Force can only control two. For a given level of the Air Force wage, the civilian wage and the probability of finding civilian employment the Air Force can choose either the expected force or the expected quality of the force. The Air Force, however, cannot select both of these variables.

As can be seen from the expression for the expected force level (42) a choice of a given quality standard determines both the arrival rate of potential enlistees and the mean length of stay in the Air Force. The mean length of stay determines the service rate which in turn determines the balking queue. Thus, the choice of a quality standard determines the balking probability and therefore the expected force level. On the other hand the Air Force can choose a desired expected force level. This desired force level can only be achieved by accepting the quality of personnel that is consistent with the force level chosen.

Under these circumstances it must be assumed that the trade off between quality and force level is made in a rational manner if its impact on the Air Force manpower market is to be understood. It seems reasonable to assume that the Air Force seeks to maximize its effectiveness. For this purpose assume the Air Force maximizes the following effectiveness function

$$E = E[\overset{+}{f}, \overset{+}{Q}, \overset{+}{(1/\mu)}] \tag{55}$$

where E is effectiveness, f, Q and μ as previously defined.

Assume that the function E is increasing and strictly concave in its arguments. The average length of stay in the Air Force affects Air Force effectiveness because it is positively related to the experience level of the force. Both the force level and the mean quality are also positively related to effectiveness. In addition, assume that while each argument has a positive influence on Air Force effectiveness the magnitude of this effect is diminishing.

As the problem is set up, the Air Force maximizes (55) by choosing f, Q and μ subject to the force level function (42) and the function for μ (22). Substituting (22) and (42) into (55) yields

$$E = E[f(W_A, W_C, P_C, Q, F), Q, \bar{\mu}(W_A, W_C, P_C, Q)] \tag{56}$$

where $f(\overset{+}{W_A}, \overset{-}{W_C}, \overset{-}{P_C}, \overset{-}{Q}, \overset{+}{F})$ is the general expression for the force level equation (42). The sign notation over the arguments of the force level equation are taken from (43) through (47). The first-order condition for a maximum of (56) is

$$E_Q = E_{(1/\mu)}\frac{1}{\mu^2}\mu Q - E_f f_Q \tag{57}$$

Condition (57) requires that the Air Force trade off force level and quality so that the marginal gain in effectiveness from an increase in quality (E_Q) exactly equals the marginal loss in effectiveness due to the reduction in mean length of stay and the force level. The problem can be illustrated via Figure 10.

The convex curves in the figure represent iso-effectiveness contours.[22] Along each of the curves the quality of personnel and the total force level are varied for fixed length of stay in the Air Force, so as to keep effectiveness constant. The further away any contour is from the origin, the greater the effectiveness.

[22]The concavity of the iso-effectiveness contours follows from the assumption of strict concavity of the effectiveness function.

The concave curve represents the force level equation for fixed length of stay in the Air Force. The point A in the figure represents the greatest attainable effectiveness given the force level equation.

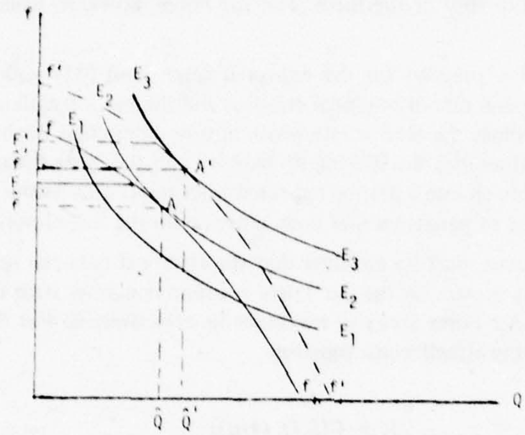

Figure 10. **Equilibrium in the force level-quality plane.**

The shape of the force level function (Figure 10) follows from the fact that there is a quality level such that potential enlistees will vanish and that even if the Air Force would take all comers, the force level *will be finite. In fact, in the latter case* $F \geq f$ *and presumably once* $F = f$ *further reductions in the quality do* not increase the force level.

Increases in the allowable force has two effects. First, it shifts outward the line ff to $f'f'$. Second, it increases the maximum force from F to F'. This change results in a move to equilibrium point A' and increases the optimal force level and quality to $\hat{f}'$ and $\hat{Q}'$, respectively.

This same device can be utilized to understand the impact of an increase in civilian opportunities. In Figure 11 the basic curves in Figure 10 have been reproduced. An increase in civilian opportunities shifts the ff curve toward the origin. That is, for any given quality of personnel the Air Force will have to settle for a smaller force. Or, alternatively, for any given force the Air Force will have to settle for lower quality. In fact, as shown in the figure, the new equilibrium is at a lower level for both the force level and quality, $\hat{f}''$ and $\hat{Q}''$, respectively.

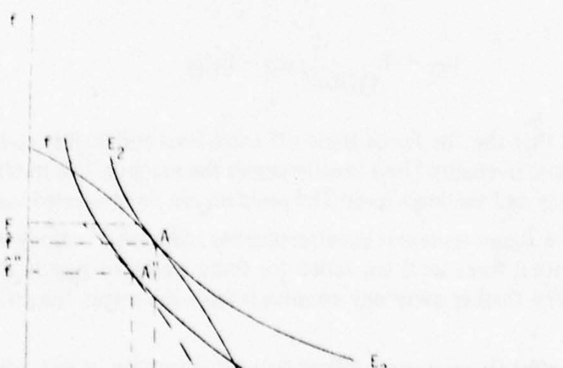

Figure 11. **Civilian opportunities and the force level.**

Up to this time the discussion has been at a level of abstraction that has little value to the Air Force. However, if it is assumed that the Air Force reaction to the changes in civilian opportunities is to maintain quality standards for re-enlistments, then the result will be increased turnover. If the Air Force also desires to maintain the force level, then quality levels of incoming personnel must be reduced. This set of circumstances would lead to increased enlistments coinciding with increased civilian wages. However, for a given quality, enlistments would have fallen.

In Section IV, the implications of the model for Air Force procurement are presented in considerably more detail than the simple example given here.

IV. THE CONNECTION BETWEEN THE THEORY AND THE AIR FORCE MANPOWER EXPERIENCE

A large number of studies have estimated the elasticity of supply of enlistees to the various military services. All of this work has strived to get results which indicate that the effect of increasing military wages is to increase enlistments. Unfortunately, the previous studies did not deal with the joint determination of the demand for and supply of enlistees, and as a result, failed to identify the supply equation. Essentially, all the studies cited have purported to be estimating supply schedules although no theoretical justification for this assumption was presented. In fact, once the retention and enlistment markets are considered simultaneously it is seen that what investigators have previously estimated are reduced form solutions. Thus, the elasticity estimates reported are not supply elasticities at all.

To see the connection between the model presented above and the identification problem consider the following graphical representation of the model. First, Figure 12 depicts the relation between expected force level and the Air Force wage (schedule ff in the figure), given civilian wages and quality of personnel. As can be seen from the figure the expected force level approaches the allowable force as the Air Force wage rises. This simply reflects the positive derivative of the expected force level with respect to the Air Force wage as reported in (43).

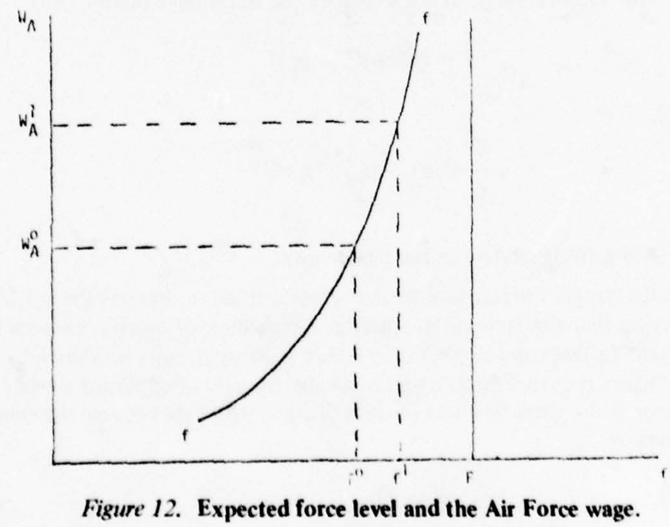

Figure 12. **Expected force level and the Air Force wage.**

For each point on ff there is associated an arrival rate of enlistees and an expected service time for existing personnel. Figure 13 shows these two schedules as they relate to the Air Force wage for fixed civilian wages, quality of personnel and allowable force level.

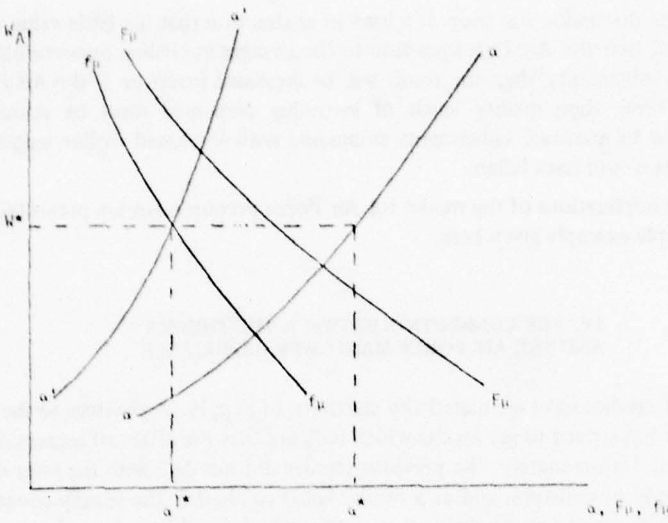

Figure 13. **Equilibrium in the accession market.**

The curve $F\mu F\mu$ relates the replacement demand for new enlistees to the allowable force level F and the mean turnover rate μ. For a given F, maintained with certainty, $F\mu$ is the average rate at which new enlistees would be needed to replace those enlisted personnel who leave the Air Force at the mean rate μ. A higher wage reduces turnover and reduces replacement demand. Thus, the Air Force replacement demand, or *demand for new accessions,* is derived from the supply of man years from the existing stock of personnel. Put another way, demand in the accession market is the complement of supply in the retention market. Recalling Figure 12, f cannot equal F but approaches it only in the limit as wages approach infinity. Thus, there is an effective accession demand curve $f\mu f\mu$, which lies below $F\mu F\mu$, reflecting the property that $f < F$ over the relevant range of Air Force wages. Recalling equation (42)

$$f = (\alpha/F\mu)(1 - p_k)F \tag{58}$$

it can be seen that

$$\frac{f}{F} = \rho(1 - p_k) = \rho' < 1 \tag{59}$$

where $\rho = \alpha/F\mu$ is the rate of arrivals relative to the service rate.

Now to construct the supply curves, assume that gross arrivals, α, exceed the service capability, $F\mu$; this is tantamount to saying that the flow of all applicants, regardless of quality, exceeds the rate at which applicants can be accepted. In this case, $\rho = \alpha/F\mu > 1$. Let gross applicants be shown by the $\alpha\alpha$ curve. As this curve indicates, the higher the Air Force wage, the greater the rate of applicant arrivals. The net flow of applicants to the Air Force is the gross flow less those applicants who balk because the wait to begin service is too long. This net flow α' is

$$\alpha' = (1 - p_k)\,\alpha \tag{60}$$

where p_k is the probability the Air Force has no current vacancies and the queue of applicants awaiting induction is at the balking value. The net arrival curve is drawn as $\alpha'\alpha'$ in Figure 13. The $\alpha'\alpha'$ lies

everywhere above and to the left of the $\alpha\alpha$ curve, reflecting the fact that at any given wage net arrivals are less than gross arrivals.[23]

At the equilibrium wage w* the $\alpha'\alpha'$ and $f\mu f\mu$ curves intersect. This reflects the steady state property that the mean throughput and arrival rates must be equal. This condition is readily derived from equation (58) by cancelling F, multiplying by μ and using the fact that $\alpha' = \alpha(1 - p_k)$ to obtain

$$\alpha' = f\mu \tag{61}$$

This equation states that the mean accession rate equals the mean force level times the mean service rate or turnover rate per position. In the steady state it is also true that

$$\frac{1}{\mu} = \frac{f}{\alpha'} \tag{62}$$

as can be readily deduced from (58). This result is simply Little's law and it indicates that the mean length of stay in the service equals the mean force level divided by the accession rate.

At the equilibrium wage w* the equilibrium accession rate α'^* equals $f^*\mu^*$, but α exceeds $F\mu$. Figure 13 is constructed to show this particular equilibrium which seems typical for the Air Force. An equilibrium does exist if α is less than or equal to $F\mu$ and has all the central properties of the equilibrium chosen here for purposes of illustration. The important point exhibited by the figure is that the process is in equilibrium when the accession market is in equilibrium; i.e., $\alpha' = F\mu$, and at this equilibrium gross arrivals need not equal the gross induction capability; i.e., $\alpha \neq F\mu$ or $\alpha = F\mu$ are possible. Thus, it is not possible to conclude that the system is in or out of equilibrium by direct observation of applicants relative to gross induction demand or recruiting guotas $F\mu$.

Some Comparative Static Implications of the Empirical Model

At this point it will prove useful to investigate the impact of changes in the various parameters which underlie the model and gain further understanding of the behavioral elements in the system and their relation to the equilibrium of the process. Here the discussion considers the impact of changes in the level of the Air Force wage, the level of wages in the competing civilian market, the authorized force level, and a measure of the employment opportunities in the civilian market.

1. *Effects of Change in the Air Force Wage.* Consider an increase in the Air Force wage from w_1 to w_2 as depicted in Panels a and b of of Figure 14. The mean force level rises in Panel a and approaches the authorized force level F. In Panel b the higher wage elicits a larger supply of applicants from α_1 to α_2. If the turnover rate μ were to stay constant, then the net supply of accessions α' would rise also along the α_1' α_1' curve. However, a higher military wage reduces μ. As a result of larger gross arrivals and smaller turnover rate, the mean force level increases and $\rho = \frac{\alpha}{F\mu}$ rises, thus leading the system to be full more often which materializes as an increase in p_k. Thus $\alpha_1'\alpha_1'$ shifts back as the gross supply α is processed into service less rapidly because of the fall in the turnover rate. Eventually, the α_1' α_1' curve shifts back to the α_2' α_2' curve which is the new equilibrium accession curve consistent with the smaller turnover rate induced by the higher wage.

At the new equilbrium the following are true; $f_2 > f_1$, $\mu_2 < \mu_1$, $f_2\mu_2 < f_1\mu_1$, $\alpha_2' < \alpha_1'$, $\alpha_1 < \alpha_2$, $F\mu_2 < F\mu_1$, $\rho_2 > \rho_1$, $p_{k_2} > p_{k_1}$. The remarkable implication of the analysis is that the new steady state induction rate α_2' is less than the old rate α_1'. Thus, an increase in the Air Force wage, while increasing the gross supply of applicants, actually reduces the accession rate because it increases personnel retention, thus lowering the rate at which accessions must occur in order to replace losses.[24]

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[23] The large distance between the $\alpha'\alpha'$ and $\alpha\alpha$ curves in the figure in no way reflects an actual situation. These curves were drawn in this manner for pedagogical purposes only.

[24] All of these conclusions are for the steady state, i.e., when the Air Force reaches a new equilibrium. In addition, if the Air Force chooses to change the quality of recruits the above impacts may be offset.

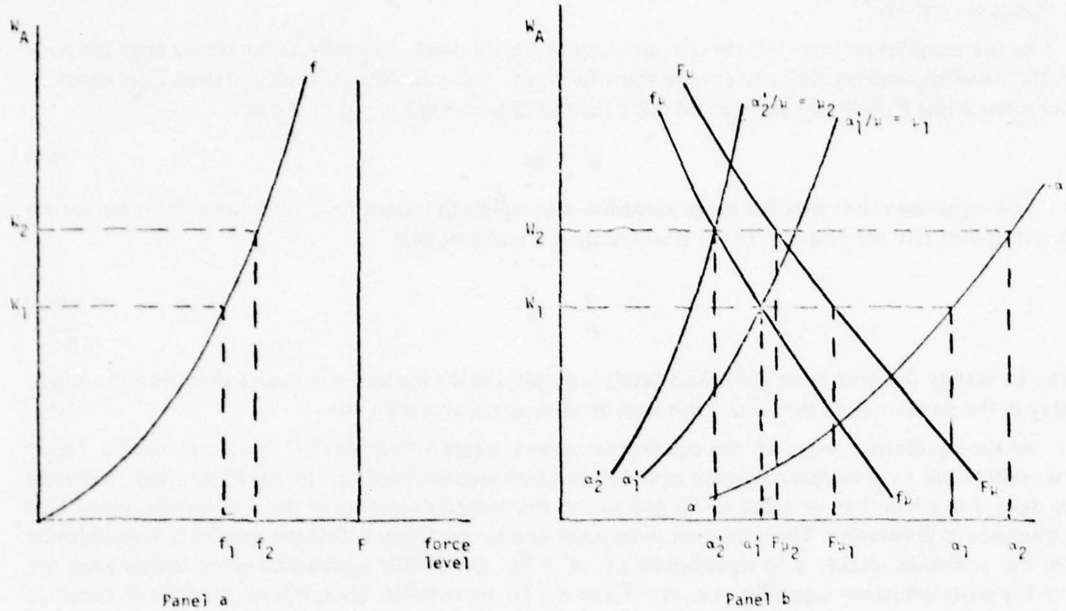

Figure 14. **The effect of changes in the Air Force wage.**

It is instructive to consider a narrower model of the accession/retention, manpower flow market. Suppose that the Air Force always meets its force level constraint exactly. To do this, the Air Force must accept applicants from the highest AFQT group compatible with a flow of accessions just adequate to maintain the force level. Such a situation is depicted in Panel a of Figure 15 where the supply curves of four AFQT groups are labelled as I, II, III, and IV. At the force level F, the number of personnel in the Air

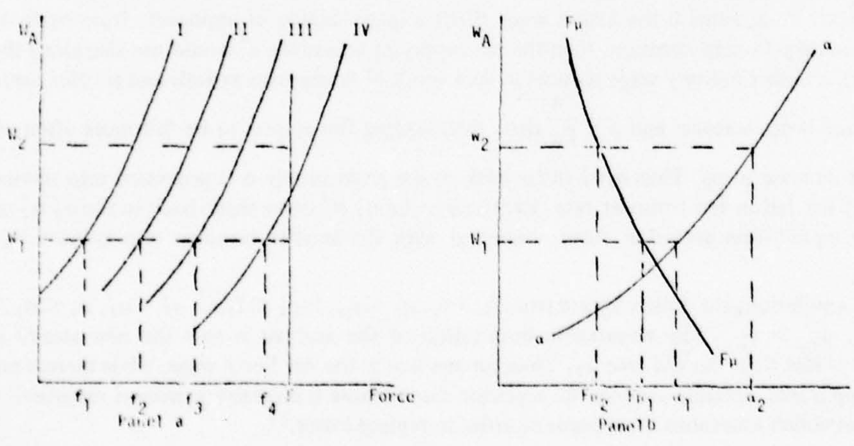

Figure 15. **Personnel quality and Air Force wages.**

Force in each mental group is read off as f_1, f_2, f_3, and f_4. In panel b the $F\mu$ curve shows turnover when the force level is always at F with the mean time in service = $1/\mu$. Now, let $\alpha\alpha$ be the total supply of potential accessions of all mental groups.[25] Then, the quantity of accessions taken by the Air Force is $F\mu = \alpha_1'$ corresponding to the wage w_1. Rejections equal the difference $\alpha_1 - \alpha_1' = \alpha_1 - F\mu_1$.

Consider now an increase in the Air Force wage from w_1 to w_2. The force level remains at F, but the new steady state mental score composition of the force, as indicated in Panel b, shows increased proportions in the higher AFQT score categories (I, II, and III relative to IV). Naturally, it will take time for this new steady state to emerge.

In the accession and retention markets the higher wage elicits an *increase* in the gross flow of potential applicants, but a *decrease* in the accession rate because of the lower personnel turnover. Clearly, with α_2 exceeding $F\mu_2 = \alpha_2'$ by a substantial amount, the Air Force can select applicants from higher AFQT categories and this eventually changes the mental composition of the force to that shown in Panel a. Augmenting this process is an ability to influence the quality of reenlisting and extending personnel by raising eligibility standards.

2. *Effects of Change in the Civilian Wage.* The analysis of this case will be facilitated by beginning with the simple certainty model wherein the Air Force maintains a fixed force level, and then extending the discussion to the more complicated stochastic model. In Panel a of Figure 16 the supplies of the four mental categories are shown at civilian wage C_1. When this wage rises to C_2, holding all other parameters of the problem fixed, these supply curves shift back from α_{11}, α_{21}, α_{31}, α_{41}, to α_{12}, α_{22}, α_{32}, α_{42}. Assuming the flows in the accession/retention market are adequate to sustain F, the force level remains unchanged, but the mental groupings are altered with higher proportions of IV's and smaller proportions of I's, II's, and III's.

In Panel b the flow supply of applicants shifts back from α_1 to α_2. In addition, retention is adversely affected and μ_1 rises to μ_2 shifting the $F\mu_1$ curve out to $F\mu_2$. In the situation depicted the Air Force enjoys a high applicant rate relative to their replacement demand and the shifts in the α and $F\mu$ curves do not put it out of equilibrium. At equilibrium, with Air Force wage w_1, the new accession rate is $\alpha_2' = F\mu_2$ which is higher than the old rate $\alpha_1 = F\mu_1$. The new applicant rate is less than the old applicant rate and the Air Force must reject a lower proportion of applicants if it is to maintain the force level.

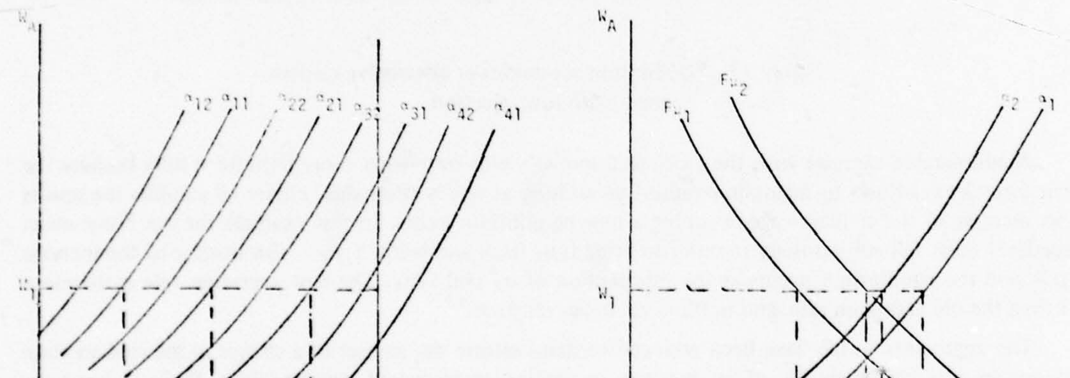

Figure 16. **Personnel quality and civilian wages: Certainty case.**

[25] The $\alpha\alpha$ curve can have two distinct interpretations. First, it can be interpreted as the arrival rate of qualified personnel so that as quality demanded declines $\alpha\alpha$ would shift right. Second, it can be interpreted as the arrival rate of all those who desire to enlist regardless of qualifications. It is this latter interpretation that is used in Figure 15.

A more extreme state of affairs might occur if the initial rejection rate is low. This situation is depicted in Figure 17. In this diagram the Air Force experiences a low rejection rate $(\alpha_1 - F\mu_1)/\alpha_1$ so that when the civilian wage rises the shifts in α and $F\mu$ take it out of equilibrium in the accession market. Applicants α_2 now are less than the flow needed to replace losses $F\mu_2$ and the force level declines. As can readily be seen, accessions decline, the rejection rate goes to zero and personnel losses exceed gains. In this case the new observed accession rate is a point on the supply curve, whereas in the former case, where a new steady state exists, the accession rate was a point on the demand curve. In the present *unstable* case a higher civilian wage *reduced* accessions because observed accessions jump from the demand curve to the supply curve. In the former *stable* case the higher civilian wage *increased* accessions because the Air Force replacement demand shifted to the right and the equilibrium point remained on the demand curve since the quantity demanded was less than offered.[26]

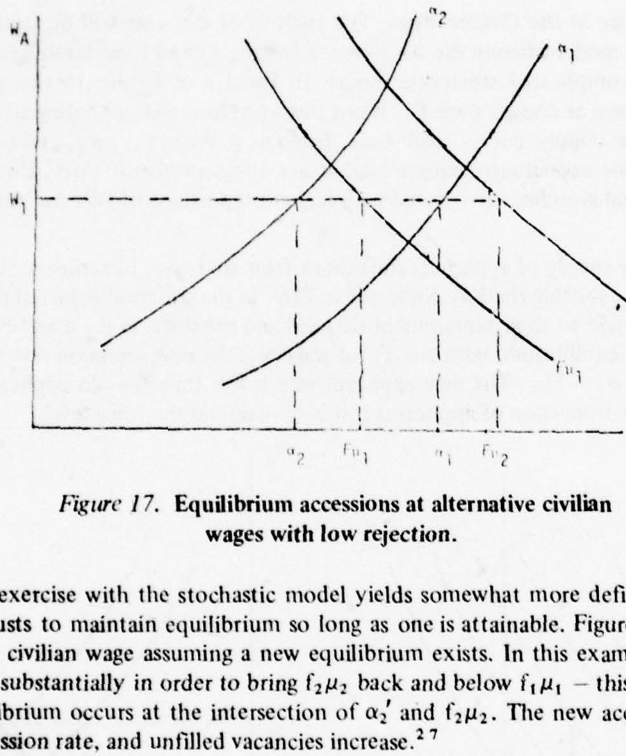

Figure 17. **Equilibrium accessions at alternative civilian wages with low rejection.**

A comparable exercise with the stochastic model yields somewhat more definite results because the mean force level adjusts to maintain equilibrium so long as one is attainable. Figure 18 exhibits the results of an increase in the civilian wage assuming a new equilibrium exists. In this example the Air Force mean force level must fall substantially in order to bring $f_2\mu_2$ back and below $f_1\mu_1$ – this in spite of the increase in μ – and the equilibrium occurs at the intersection of α_2' and $f_2\mu_2$. The new accession rate is therefore *less* than the old accession rate, and unfilled vacancies increase.[27]

The arguments which have been adduced to demonstrate the impact of a change in the civilian wage also convey the major results of an increase in civilian employment opportunities. While it has been conventional to use the unemployment rate U or its complement (1 – U) as a measure of weakness or strength in the civilian labor market, it seems more reasonable to use the labor force participation rate. Whatever measure is used, it can be seen that the model predicts a non-conventional result for changes in

[26] It is just this shifting of the equilibrium number of accessions from the demand schedule to the supply schedule depending on market conditions that results in the lack of identification of the previously estimated supply schedules.

[27] To understand this seemingly anomalous result it must be kept in mind that the increase in civilian wages reduces the mean force level for given quality. Accordingly, the turnover rate at the new lower mean force level is lower. It does not follow that rejections increase. In fact, they will, in general decline.

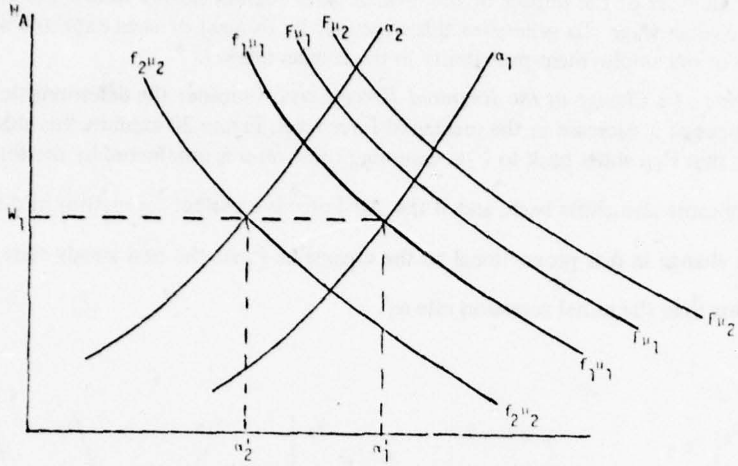

Figure 18. **Accession rates and civilian wages: The stochastic case.**

the strength of the civilian labor market. Suppose, for example, that the conventional view was adopted so that an increase in the unemployment rate signified a decrease in civilian employment prospects. Under this conventional view accessions are on the supply curve and an increase in accessions should occur when unemployment rises. In the present model this is not the case. First an increase in the unemployment rate increases retention thus shifting the $F\mu_1$ curve back to $F\mu_2$. Accompanying this change is a shift outward in the accession supply curve from α_1 to α_2. This is shown for the certainty model in Figure 19. The new equilibrium accession rate is smaller than the old rate, not larger. The Air Force rejection rate rises and the

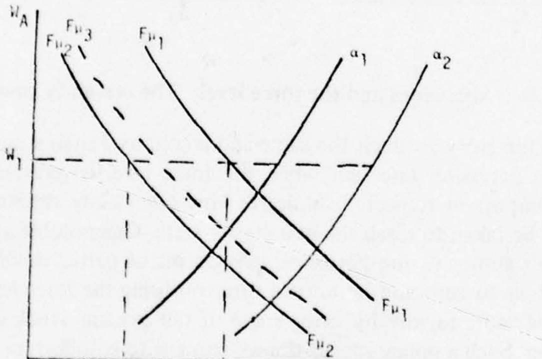

Figure 19. **Accessions and the strength of the civilian labor market.**

mental score composition will tend to rise as standards are raised. The movement of observed accessions α_1' to α_2' actually reveals a supply elasticity for extensions and reenlistments rather than any supply behavior in the accession market. It seems reasonable that in light of the better retention supply the Air Force might declare a slightly larger fraction ineligible for reenlistment thereby improving the AFQT distribution on both ends of the experience distribution. This would be revealed as an increase in μ so that the final shift in μ might go from μ_1 to μ_3 rather than to μ_2. Of course, any skimming of the AFQT distribution at the retention market reduces the skimming that may be done in the accession market.

Further analyses of the impact of the civilian labor market simply follow the lines indicated for the effect of the civilian wage. To generalize this wage may be thought of as an expected wage pWc, where p is some measure of the employment probability in the civilian market.[28]

3. *Effects of a Change in the Required Force Level.* Consider the deterministic model and examine the consequences of a decrease in the mandated force level. Figure 20 exhibits this change with F_1 shifting back to F_2 so that $F_1\mu$ shifts back to $F_2\mu$, assuming turnover μ is unaffected by the force level change. The supply of applicants also shifts back, and if the Air Force is a perfect competitor in the labor market, $\frac{\partial\alpha}{\partial F} = \frac{\alpha}{F}$ so that the change in α is proportional to the change in F. At the new steady state, accessions α_2' equal $F_2\mu$ and are less than the initial accession rate α_1'.

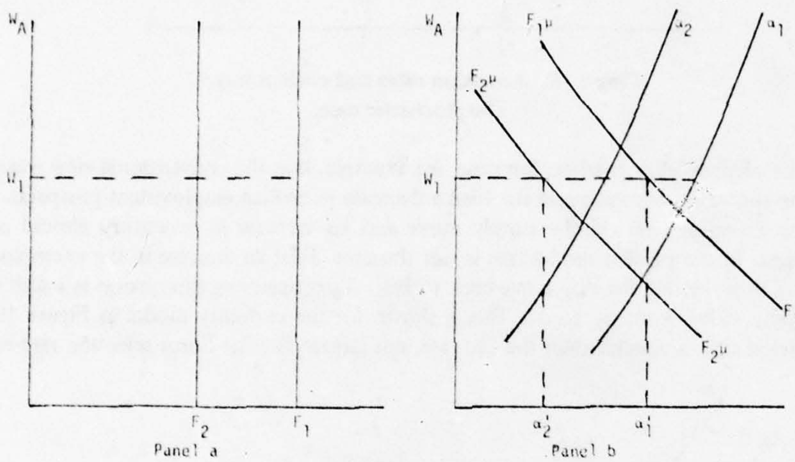

Figure 20. **Accessions and the force level: The certainty case.**

In the stochastic model the story is much the same and is told by Panels a and b of Figure 21. Again the conclusion is that the net accession rate falls when the force level is reduced. This description is a simplification in at least one important respect. It indicates how one steady state compares with another, but ignores the path that may be taken to reach the new steady state. One notable aspect of the adjustment process may be the Air Force's ability to fire personnel as a means of paring down the force level. Thus, rather than shut down accessions to zero and let normal turnover bring the force level down to the desired size, the target can be achieved more rapidly by firing some of the existing stock of personnel, by cutting reenlistment bonuses and so on. Such a policy speeds the adjustment to equilibrium and prevents shutdown of the training schools, recruiting offices and other parts of the accession pipeline, which would be required if accessions were to be frozen at zero until the required lower force level is achieved. Moreover, if the Air Force selectively fires to speed the movement to the new equilibrium, it might be expected that the AFQT distribution of the enlisted personnel in the Air Force would improve.

[28] Both the long run and short run employment opportunity in the civilian markets are relevant for the enlistment and desired length of stay decisions. The short run rate results in a cyclical effect on retentions and arrival rates. The net effect of this cyclical effect depends on whether or not the Air Force uses a cyclical quality standard. At constant quality, as pointed out above, the level of accessions will be inversely related to unemployment.

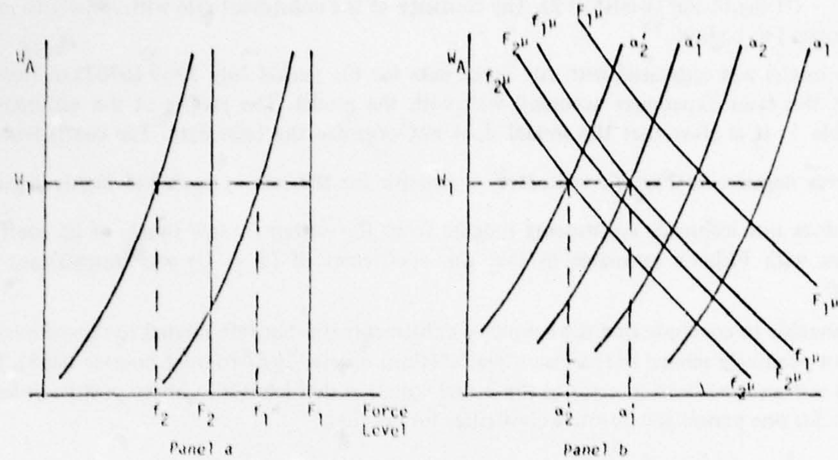

Figure 21. **Accessions and the force level: The stochastic case.**

V. A PRELUDE TO THE EMPIRICAL WORK

Before developing the detailed empirical model, to be estimated, it may be advisable to establish the empirical relevance of the approach taken here by examining its implications for earlier studies of military manpower supply. Because the model discussed here links the retention and accession markets and incorporates quality rationing (by AFQT category), it has significant implications for these earlier studies that did not consider these aspects of the military manpower market.

Earlier studies have focused on either the accession market or the retention market, but have not linked them together in a simultaneous system. In the accession studies, the focus has been on estimating the supply function for new accessions which relates the accession rate to the military wage, the civilian wage and other determinants of labor supply such as the unemployment rate. From the preceding section it should be clear that there are serious doubts as to whether these studies have identified a supply function at all owing to the fact that the accession rate is related to supply *and demand*. There are essentially three issues that must be addressed in an examination of this work: (a) Is the supply function identified? (b) What analytical treatment is accorded to quality? (c) How do the models stand up to the recent experience with Air Force enlisted personnel? In this discussion, attention is confined to two studies, one using time-series data and the other using cross-sectional data, since there are no substantive differences in the models specified in the other studies of military manpower supply.

The Fisher Study

In a respected study, Fisher (1969) examined the relation:

$$\frac{E}{P} = \alpha_0 + \alpha_1 \ln(\frac{Wc}{Wm}) + \alpha_2 \ln(1 - U) + \alpha_3 \ln(1 - \frac{A}{P}) + \alpha_4 SP + \alpha_5 SU + \alpha_6 AU \qquad (63)$$

where E = total Armed Forces enlistments in mental categories I, II, and III, P = male civilian population age 17 through 20, Wc = civilian wage, W_M = military wage, U = unemployment rate, males age 18 through 19, A = total accessions (enlistments plus drafts), and ln indicates the logarithm of the variable. Seasonal dummy variables for spring, summer and autumn were also included and are, respectively, SP = spring, SU =

summer and AU = Autumn. Fisher found $\alpha_1 < 0, \alpha_2 < 0, \alpha_3 < 0, \alpha_4 < 0, \alpha_5 > 0, \alpha_6 < 0$, with all but the coefficient of $(1 - U)$ significant (t-ratio ≥ 2). The elasticity of the enlistment rate with respect to military earnings was estimated to be 0.46.[29]

The Fisher model was estimated with Air Force data for the period July 1969 (6907) to June 1976 (7606) to see if this later experience accorded well with the model. The results of the estimation are presented in Table 1. It is clear that the model does not organize the later data. The coefficient α_1 of $\ln\frac{Wc}{Wm}$, which was negative in Fisher's estimates, is positive for the later period and highly significant. Unemployment does not influence enlistments judging from the extremely low t-ratio of its coefficient. This result agrees with Fisher's estimates in that the coefficient of $(1 - U)$ was insignificant in his regressions as well.

It is unreasonable to conclude that the supply of enlistments is negatively related to the military wage from 1969 on but positively related in the Fisher period (third quarter 1957 to third quarter 1965). Rather there is clearly a serious specification error in the Fisher equation that leads it to imply positive enlistment supply elasticities for one period and negative elasticities for another.

Table 1. **Estimates of Fisher Model with Air Force Enlisted Personnel Data for the Period 6907 to 7606**

C	$\ln\frac{Wc}{Wm}$	$\ln(1-U)$	SP	SU	AU	R^2
−.0003	.0008	−.0002				.76
(3.36)	(15.47)	(0.28)				
−.0003	.0008	−.00006	−.00001	.00002	0	.77
(2.66)	(14.33)	(0.07)	(0.27)	(0.42)	(0.09)	

The model developed here indicates that observed enlistments may be on the supply curve or the demand curve depending upon the wage, force level requirements, and supply of applicants. The Fisher equation must be considered to be a reduced form equation rather than a supply equation and therefore is not indicative of the supply response of enlistees to alternative wages. This comment also applies to the other supply studies reviewed; they have failed to identify the supply curve and have instead estimated a reduced form equation of a simultaneous equations model.

The explanation for Fisher's results with the context of the model developed here would be as follows: let w_1, referring to Figure (22), be the ratio of the military wage to the civilian wage at the early part of the time period. At this wage and given the force level requirement F, the replacement demand of the Armed Forces is $F\mu(w_1)$, where the turnover rate μ is a function of the wage ratio with a negative first derivative. The ratio w drifted downward over the period according to Fisher. So let w_2 represent the later wage ratio. At w_1 the supply of volunteer enlistments is α_1 and the number drafted is $d = F_1\mu(w_1) - \alpha_1$. In Fisher's equation there is a positive relation between the proportion of accessions drafted and enlistments, which he interprets as arising from draft pressure. In order to get this relation it is necessary to assume that the force level was reduced, say, from F_1 to F_2. This results in the positive relation between the wage and supply, as shown by points on the supply curve, as well as a positive relation between the draft rate and enlistments as indicated by a comparison of (α_1, d_1) with (α_2, d_2). If there were draft-induced enlistments, this would be shown as a rightward shift of α to a position between α_1 and α_2 such as shown by the α^d curves.[30]

[29] Although issue is taken with Fisher concerning the meaning of his estimates the overall piece of work was outstanding in its understanding of the data and the military accession problem.

[30] The α^d curves meet the α schedule at the replacement demand since at wages above this level no drafting occurs so that draft induced enlistments vanish.

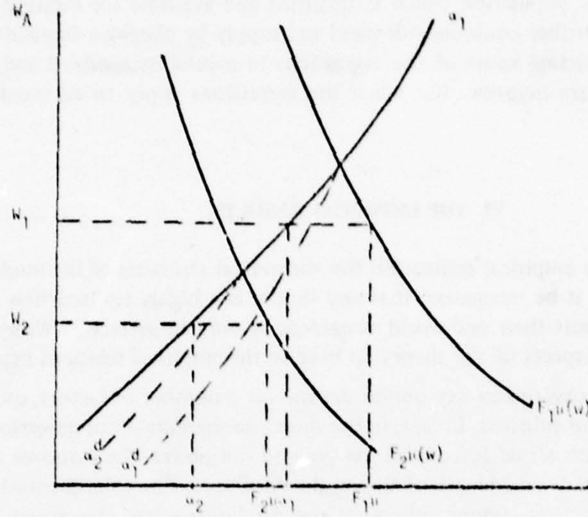

Figure 22. **Fisher's "supply" estimate.**

In the absence of draft-pressured enlistments, the Fisher equation would estimate the wage supply elasticity, at least insofar as it would not be confounded by demand effects of the period analyzed. In the later sample period, however, this is not the case as indicated by the estimates presented in Table 1. These estimates show a negative relation between the military/civilian wage ratio and enlistments. This result is easily explained by the reduction in the force level and rise in the wage ratio which characterizes this later period. As Figure 23 indicates, the enlistment rates are not points on the supply curve but are points on shifting demand curves instead.

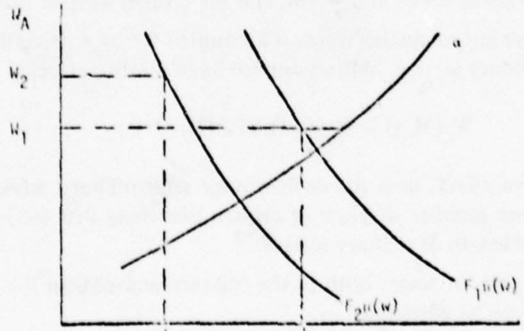

Figure 23. **Wage enlistment relation in the post AVF period.**

Grissmer et al

A more recent study of Air Force enlistments by Grissmer et al. (1974) is also plagued by identification problems. Of the five regressions reported, two had a negative coefficient for the military/civilian wage ratio. The model used is a variant of the Fisher model.

These regressions are estimated with cross-sectional rather than time-series data, but it is clear that problems similar to those encountered in time-series also appear and make it unclear whether the

enlistments observed are points on the demand or supply curves. The practice of restricting the measurement of supply to the population which is qualified and available for military service (QMA) as determined by AFQT scores further confounds demand and supply by placing a demand-related constraint on supply. For example, restricting some of the regressions to mental categories I and II, give "supply" regression coefficients which are negative. But when the regressions apply to all mental categories, the coefficients are positive.

VI. THE EMPIRICAL RESULTS

Before proceeding to the empirical estimation the theoretical structure of the model must be related to the data. This requires that it be recognized that any theory is a highly stylized view of the world. The elements of the theory represent their real-world counterparts only imperfectly. With this in mind this section will bring the various aspects of the theory to bear on the empirical relations expected in the data.

The actual number of accessions in any period depend on a number of factors, only one of which is the rate of arrival of prospective enlistees. In fact, in the short run the number of accessions must equal the turnover plus the change in the actual force level. As pointed out above, the turnover rate in the steady state is determined by the mean desired length of stay in the Air Force. From the theoretical discussion this desired length of stay is a function—among others—of the Air Force wage, the civilian wage and civilian employment opportunities. Thus, the demand for and the supply of accessions are not independent.

The arrival rate of potential enlistees depends on the number of individuals who have desired Air Force stays large enough to make the minimum enlistment superior to a working career with no Air Force service. As shown above, the desired years of military service for any given permanent consumption stream depends on military and civilian wages. This relation can be written as

$$PV = \int_{O}^{M} e^{-rt} W_m(t)dt + \int_{M}^{M+C} e^{-rt} W_c(M, t)dt \qquad (64)$$

where PV is present value, r is the interest rate, M and C are respectively years in the military and years as a civilian, $W_m(t)$ is the military wage at time t and $W_c(M, t)$ is the civilian wage at time t.[31]

The effect of military training on civilian wages is accounted for by writing the civilian wage at time t as a function of the years of military service. Military service has a positive effect if

$$W_c(M, t) > W_c(O, t) \; \forall t \in (M, M+C) \qquad (65)$$

That is, if training has a positive effect, then the civilian wage after military service will be higher than it would have been after the same number of years of civilian life. Note that (65) is neither necessary nor sufficient for a non-zero desired length of military service.[32]

If it is assumed that the path of wages both in the military and civilian life grow exponentially at a constant rate, then both wages can be written as

$$W_m(t) = W_m(O)e^{\phi t} \qquad (66)$$

---

[31] The form of (64) shows clearly the distinction between our approach and the career choice approach. In the career choice approach the relevant comparison would be

$$\frac{PV_m}{PV_c} = \frac{\int_O^{L_m} e^{-rt} W_m(t)dt}{\int_O^{L_c} e^{-rt} W_c(t)dt}$$

so that combinations of military and civilian work effort are ignored.

[32] That is preferences for the military versus civilian life also enter the picture.

$$W_c(M, t) = W_c(M, M)e^{\Theta(t-M)} \tag{67}$$

From (67) it follows that (65) holds if

$$W_c(M, M) > W_c(O)e^{\Theta M} \tag{68}$$

Now letting $W_c(M, M)$ be proportional to $W_c(O)$ yields

$$W_c(M, M) = \psi(M)W_c(O)e^{\Theta M} \tag{69}$$

where $\psi(M) > 0$ is the factor of proportionality. Note that if military training has no civilian value, then $\psi(M) = e^{-\Theta M}$ and $W_c(M, M) = W_c(O)$. Using (69) write (67) as

$$W_c(M, t) = \psi(M)W_c(O)^{\Theta t} \tag{70}$$

Using (66) and (70) rewrite the present value of total lifetime earnings as

$$PV = \int_O^M e^{(\phi-r)t}W_m(O)dt + \int_M^{M+C} e^{(\Theta-r)t}\psi(M)W_c(O)dt \tag{71}$$

which is a function only of beginning military and civilian wages. From (71) the slope of the iso-present value constraint (the constant permanent consumption locus) is

$$\frac{\partial M}{\partial C} = -\frac{\frac{W_c(O)}{W_m(O)}e^{(\Theta-r)(M+C)}}{\psi(M)e^{(\phi-r)M} + \frac{W_c(O)}{W_m(O)}e^{(\Theta-r)M}[e^{(\Theta-r)C} - 1]} < 0 \tag{72}$$

Which is negative and a function of the relative wage. In fact, the greater the civilian wage is relative to the military wage, the steeper is the iso-consumption locus since

$$\frac{\partial(\frac{\partial M}{\partial C})}{\partial\left[\frac{W_c(O)}{W_m(O)}\right]} = -\frac{\psi(M)e^{(\phi-r)M}e^{(\Theta-r)(M+C)}}{\left\{\psi(M)e^{(\phi-r)M} + \frac{W_c(O)}{W_m(O)}e^{(\Theta-r)M}(e^{(\Theta-r)C} - 1)\right\}} \tag{73}$$

is definitely negative.[33]

Clearly then the likelihood that any individual will have a desired military stay sufficiently large to induce enlistment for a given minimum enlistment period is positively related to the military beginning wage and negatively related to the beginning civilian wage. Alternatively, this likelihood is positively related to the ratio of beginning military to beginning civilian wages. Given the likelihood of enlistment the arrival rate of potential enlistees depends on the number of individuals at the appropriate point in their work careers. The number of individuals in the 16 through 19 age group is usually taken to be the relevant group for this case.

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[33] As is clear from (72) this iso-present value constraint is not linear in M and C. In fact, it is concave. This concavity of the constraint in no way affects our analysis so long as the constraint is less concave than the indifference curves.

In addition to the relative beginning wages the probability of being employed has been ignored in the above analysis. In the long run however, a reduction in the probability of being employed will increase the likelihood that an individual with a positive desired military service that is less than the minimum required will nonetheless find it optimal to enlist. Thus, some measure of the probability of being employed in the civilian sector must be used in explaining short-run changes in the arrival rate.

In addition to the impact of relative wages on the arrival rate the theory has shown the importance of their impact on the desired years of military service. This latter element is important because length of service along with the allowable force determines the turnover rate and thus, the demand for enlistees. In the Air Force market the actual number of enlistees is determined jointly by the arrival rate (supply) and the force level maintenance requirement (demand). This latter element is composed of the mean length of stay in the Air Force and the allowable force level. What is observed as enlistments will, when force level maintenance requirements exceed arrivals, be a point on the supply curve and, when arrivals exceed force level maintenance requirements, be on the demand curve.

The following paragraphs contain a brief description of the data used for the statistical analysis. The data description accounts for the fact that individuals enter the decision making group rather unevenly through the year resulting in a seasonal effect on the arrival rate. In addition, during some of the period of the data the draft was still in effect. In this case the decision making process described earlier must be modified since even those with zero desired military service may find it optimal to enlist. In both of these cases dummy variables were used to account for these effects.

Description of Data

This section presents a description of the variables employed in the analysis. It is of particular importance to note certain institutional aspects of the Air Force personnel system which have generated the available data. Initially, several alternative measures of first-term enlistments are discussed. This is followed by a delineation of the nature and sources of the data which appear as explanatory variables.

Much of the empirical work which follows is intended to explain observed variations in accessions of Air Force enlisted personnel. There are two important dates associated with each enlistment: "Total Active Federal Military Service Date" (TAFMSD, or service date) and "paydate." The latter refers to the date on which the individual executes an enlistment contract, that is, the date on which longevity begins accruing for pay purposes. TAFMSD is the date on which the enlistee actually commences active duty. Supply-related factors such as seasonality in enlistments and demand-related constraints such as end-strength requirements result in the fact that, for some individuals, paydate precedes TAFMSD. For example, a high school senior might decide to enlist in January, but not to report for active duty until after graduation in June. Or, to prevent the size of the enlisted force from exceeding Congressionally-mandated manpower levels near the end of a particular fiscal year, some enlistees may be "carried over" to the following year. In any case, these individuals are assigned to the inactive Reserve for the period during which they are waiting to begin active service.

The above considerations give rise to three possible measures of the gross monthly flow of enlistees to the Air Force: enlistments based on paydate, enlistments based on TAFMSD, and enlistments of individuals for whom paydate and TAFMSD coincide.

On *a priori* grounds, one would expect enlistments based on paydate to exhibit a relatively larger degree of fluctuation over time than the other two measures. Seasonal trends would be expected to be more pronounced in the paydate enlistment series since this represents the unadjusted monthly number of "hires" by the Air Force.

If the Air Force attempts to "smooth" the flow of enlistees to the active force, this should be reflected in the TAFMSD data. Enlistments based on service date give a measure of the number of individuals who report to begin employment each month.

Finally, the enlistment data on individuals for whom paydate and TAFMSD coincide can be interpreted as representing "immediate enlistments." These are enlistees who do not wait to begin employment and this series would, in general, seem to follow the basic hiring pattern of the Air Force.

Figure 24 displays the three enlistment series for the entire time period covered by the present analysis. The data were derived from a computer tape made available by the Air Force Human Resources Laboratory.

Data relating to the civilian labor force were obtained from published and unpublished sources of the Bureau of Labor Statistics (BLS).[34] It was assumed that the relevant group from which potential enlistees are obtained is the total male, noninstitutionalized population, ages 16 to 19 years, inclusive. No attempt has been made to adjust this group to obtain an estimate of the so-called "qualified military available" (QMA). For this age-sex classification, monthly observations were gathered on the following variables: total civilian labor force participants, total not in the labor force, and total unemployed. In accordance with BLS definitions, the total non-institutionalized population of 16 through 19-year-old males is the sum of those in the labor force and those not in the labor force. The unemployment rate is, then, the total unemployed divided by the number in the labor force. The employment rate appearing in some of the regressions is calculated to be unity minus the unemployment rate.

In the initial regressions, civilian wages were taken to be the average weekly earnings of production or nonsupervisory personnel on private nonagricultural payrolls. These data were multiplied by 4 to obtain monthly wage equivalents. However, no adjustments were made to take account of variations in the length of the work week. In these same regressions, military wages are the monthly basic compensation of individuals in paygrade E-1. Although enlisted personnel receive a variety of other allowances both directly and in kind, most forms of compensation are proportional to base pay so that using only the latter as a measure of military wages affects absolute, rather than relative, levels of militay pay.

Over a period covered by this study, the enlisted force declined from a strength of almost 725,000 at the end of FY 1969 to just over 480,000 by the close of FY 1976. To take account of this reduction, a force level variable appears in many of the regressions: it is measured by entering the end-of-fiscal-year manpower ceilings established in the Congressional budgeting process. Over the same period, however, the number of individuals in paygrade E-1 rose from 18,000 at the end of FY 1969 to a peak of just over 25,000 in FY 1973, then declined to about 20,000 at the end of FY 1976. This trend is reflected in variable "E1."[35]

Finally, four dummy variables have been introduced. Three are seasonal dummies: the spring dummy (SPDUM) has the value of 1 for the months of March, April, and May; the summer dummy (SUDUM) equals 1 for June, July, and August; and the autumn dummy (AUDUM) has the value 1 for September, October, and November. To take account of the possibility that the operation of the Selective Service System had an impact on Air Force enlistments, a draft dummy (DFTDUM) was entered. The draft dummy has a value of 1 for January 1973, or earlier, and zero otherwise.[36]

Table 2 summarizes the major points discussed in this section.

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[34] U.S. Department of Labor, Bureau of Labor Statistics. *Employment and Earnings*. Washington, D.C.: U.S. Government Printing Office, various issues.

[35] Force level inventories are depicted in Appendix D.

[36] President Richard Nixon announced the end of the draft on January 27, 1973. The Air Force is the only one of the four services which has never used the draft to procure enlisted personnel, so the DFTDUM can be interpreted as representing an exogenous inducement to enlist in the Air Force rather than the U.S. Army.

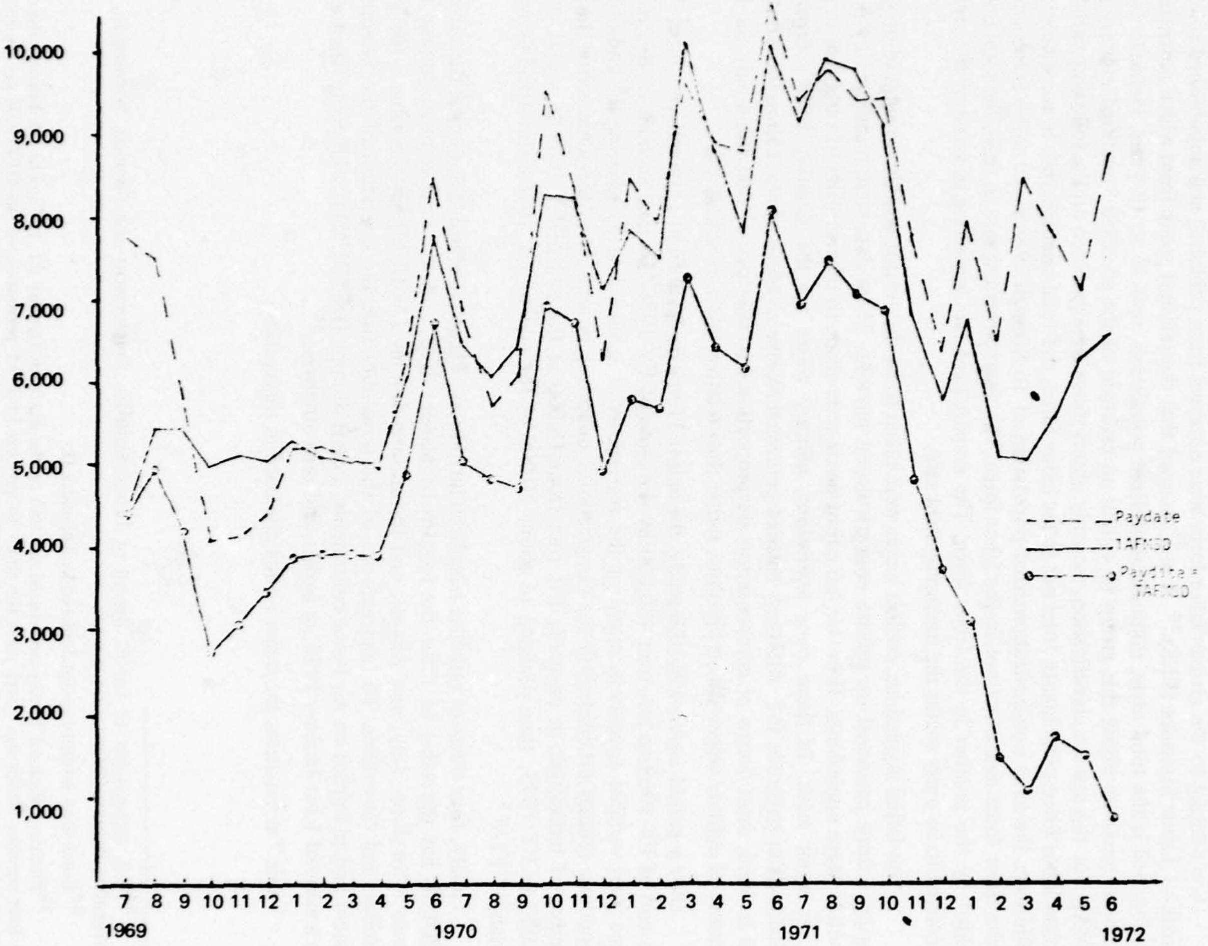

Figure 24. **Gross enlistments.**

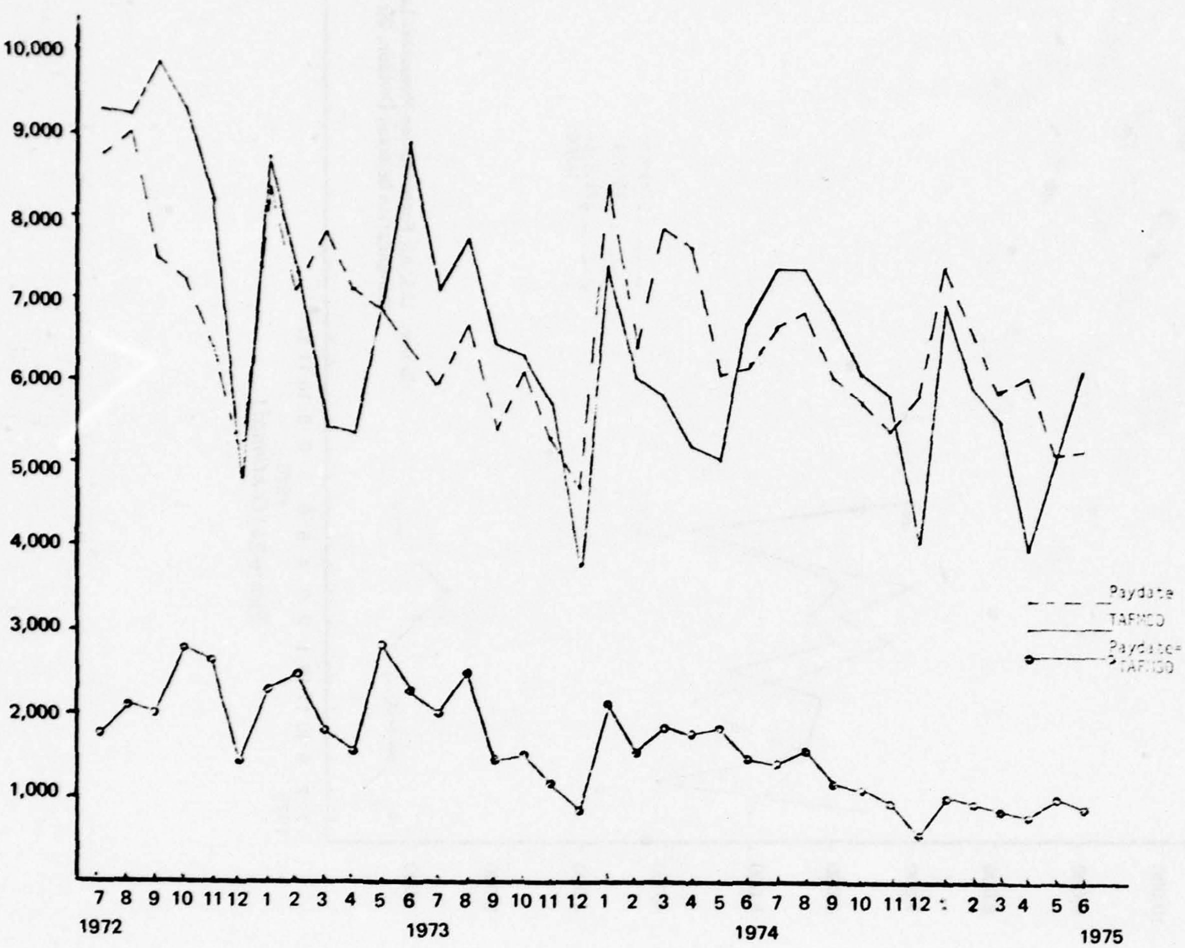

Figure 24 (*Continued*)

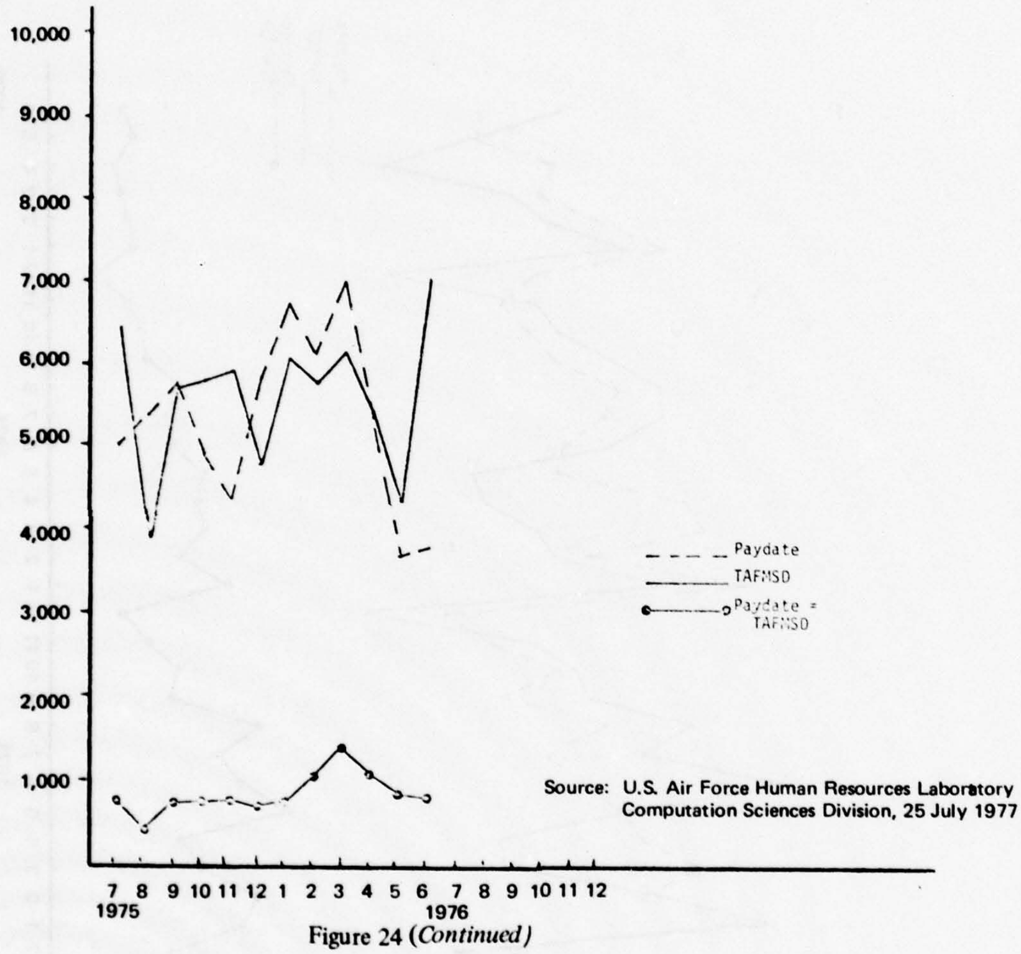

Figure 24 (*Continued*)

Table 2. **Summary of Variable Definitions and Sources**

Variable Name	Description	Source
ENLIST	Gross enlistments based on: 1. paydate 2. TAFMSD 3. paydate = TAFMSD (see specific regression for definition used)	USAFHRL
ERATE	Enlistment rate. Gross enlistments divided by total population of 16–19-year-old, non-institutionalized males.	USAFHRL and BLS
U	Unemployment rate. Total unemployed divided by total labor force participants in relevant population group.	BLS
EMP	Employment rate. 1-U	BLS
W^C	Civilian wages. Average weekly earnings of production or nonsupervisory personnel on private nonagricultural payrolls times 4.	BLS
W^M	Military wages. Monthly base pay for grade E1.	USAFHRL
W^R	Wage ratio. W^M/W^C	USAFHRL and BLS
POP	Total population of 16–19-year-old, noninstitutionalized males.	BLS
LFPR	Labor force participation rate. Total labor force participants (employed plus unemployed) divided by population in relevant age-sex group.	BLS
FORL	Force level. Congressionally-mandated end strength (total enlisted force).	USAFHRL
E1	Inventory of total individuals in paygrade E-1.	USAFHRL
SPDUM	Spring Dummy 1 = March, April, May 0 = otherwise	
SUDUM	Summer Dummy. 1 = June, July, August 0 = otherwise	
AUDUM	Autumn Dummy 1 = September, October, November 0 = otherwise	
DFTDUM	Draft Dummy 1 = January 1973 or earlier 0 = otherwise	

A Model for Empirical Estimation

As a means of deriving an empirical model suitable for estimation consider the basic steady state equation derived in Section IV.

$$\alpha' = \rho(1 - p_K)F\mu. \tag{74}$$

Let $\pi = 1 - p_K$ and use the fact that $f = \rho\pi F$ to write (74) in two different forms

$$\alpha' = f\mu \tag{75}$$

and

$$\alpha' = \rho\pi F\mu \tag{76}$$

In order to estimate these equations the levels of f, ρ, π and μ must be related to the exogenously determined variables W_m, W_c and F.

Begin with (75) and assume that

$$f = \phi\, W_m^{f_1}\, W_c^{f_2}\, F^{f_3} \tag{77}$$

where the f_i are the elasticities of the mean force level f with respect to the i-th exogeneous variable. In Section III, it was shown that $f_1 > 0$, $f_2 < 0$ and $f_3 > 0$. Now, specify the turnover function in a similar fashion, viz.

$$\mu = \Theta\, W_m^{\mu_1}\, W_c^{\mu_2} \tag{78}$$

where the elasticities of average turnover are $\mu_1 < 0$ and $\mu_2 > 0$. Then (75) may be written

$$\alpha' = \phi\, W_m^{f_1}\, W_c^{f_2} F^{f_3}\, \Theta W_m^{\mu_1} W_c^{\mu_2} \tag{79}$$

or taking logs

$$\ln\alpha' = \ln(\phi\Theta) + (f_1 + \mu_1)\ln W_m + (f_2 + \mu_2)\ln W_c + \mu_2 \ln F \tag{80}$$

This equation relates the mean accession rate to the starting military and civilian wages and the force level requirement. Note that (80) is a reduced form equation and information about the structural relationships, say the supply equation, is given only indirectly from estimates of (80). In fact the elasticity of accessions, α', with respect to the military wage is $f_1 + \hat{\mu}_1$. This is the sum of a positive term, f_1 = the elasticity of mean force level with respect to the military wage, and a negative term, μ_1 = the elasticity of turnover or the inverse of time in service with respect to the military wage. In a regression of enlistments against W_m, W_c and F, the coefficients of W_m do not indicate a supply elasticity. Rather, what is obtained in such a regression, is $f_1 + \mu_1$ which is the *sum* of two opposite forces, one a tendency for higher military wages to increase the mean force level because of a greater forthcoming supply of volunteers, another a tendency for the demand for accessions to decline because higher wages increase retention of personnel.[37]

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[37] It is quite possible that a regression estimate of (80) would yield a zero coefficient for $\ln W_m$ even though both f_1 and μ_1 are significantly non-zero.

This important finding has profound implications for the feasibility of an all volunteer armed force because it indicates that both of the fundamental supply elasticities – to the accession and retention market–have not yet been correctly estimated and moreover, the enlistment supply elasticity has been greatly underestimated.

Equation (80) is to be estimated in the next section. Before doing so the unemployment rate must be put in the equation. This variable has attracted a good deal of attention in the literature on military manpower, but its treatment has been without adequate theoretical foundation. It is clear (from the discussion in Section II) that the unemployment rate primarily influences short-term enlistments. The reason for this is that fluctuations in the unemployment rate are common and the rate is expected to return to some normal level. Thus, the present values of careers in the civilian or military sectors are not strongly affected by variations in unemployment. What may be affected, however, is the fraction of job searchers who are willing to enter the military for a short period of time – individuals whose optimal career profile would have otherwise included no time in the military.[38] Further, a temporary high level of unemployment would induce those presently in the service to extend rather than leave at the planned time. As a consequence, the effect of unemployment on accessions is a netting out of two offsetting changes. High unemployment increases the fraction of individuals joining the military, but by more for those planning short stays. High unemployment also induces those presently in to extend, thus reducing accession demand. The net affect on observed rates depends on the magnitude of these offsetting effects.

Let supply be a function of the "expected" civilian wage $(1 - U)W_c$, where U is the unemployment rate; then it is readily shown that equation (80) may be rewritten as

$$\ln\alpha' = \ln\phi\Theta + (f_1 + \mu_1)\ln W_m + (f_2 + \mu_2)\ln W_c + (f_4 + \mu_3)\ln(1 - U) + f_3 \ln F. \tag{81}$$

In estimable form this equation becomes

$$\ln\alpha' = \beta_0 + \beta_1 \ln W_{mt} + \beta_2 \ln W_{ct} + \beta_3 \ln(1 - U) + \beta_4 \ln F_t + \epsilon_t \tag{82}$$

where ϵ_t is a normally distributed random error term with zero mean, and t refers to the time of the observation (monthly in the sample used). A test of the hypothesis that unemployment enters the enlistment/extension decision as though it were the probability of securing the wage W_c is whether or not $\beta_2 = \beta_3$. If they are not equal, then unemployment enters the supply/retention process in some other fashion. Generally β_2 should not equal β_3 because different unemployment rates apply to enlistments and continuations and these decisions involve different age–experience groups.

One other equation to be estimated is another form of (80) which uses the ratio W_m/W_c. This equation may be written

$$\ln\alpha' = \ln\phi\Theta + (f_r + m_r)\ln\left(\frac{W_m}{W_c}\right) - (f_r + \mu_r)\ln(1 - U) + f_3 \ln F \tag{83}$$

Here f_r, μ_r are the elasticities of f, and μ with respect to the wage *ratio*. This equation contains no information which differs from (80). It is used because of the large number of studies that have used the wage ratio.

Estimates

Estimates of (82) and (83) are obtained using three different measures of the accession rate as explained in Section VI. These estimates are discussed in order beginning with accessions whose paydate and service dates are equal, then moving to accessions by paydate and then by service date.

[38]The result of a temporarily high level of unemployment is to make the constraint convex to the origin. As a result, as shown in Section II, the optimal level of military service increases so that arrivals of potential enlistees will increase.

Accessions: Paydate = Service Date

An accession whose pay and service dates are equal is an individual who commenced active duty on the date he began accruing longevity. This individual may be regarded as one who neither elected to nor was required to wait to begin service after executing his enlistment contract. In order for such an event to occur, there must be a vacancy in the force when the individual applies for enlistment. It follows then that this individual did not wait in a queue of applicants, but was processed right into employment upon his "arrival" or his execution of the contract. The proportion of individuals who are able to begin employment immediately is related to the proportion of the time the force is below its authorized level. This in turn is related to $\rho' = f/F$ or the ratio of the mean to the authorized force level

Thus whatever *increases* the proportion of time the service is at full strength, as measured by ρ', should *reduce* accessions whose pay and service dates match. This hypothesis is broadly confirmed by the data in Appendix C as well as Figure 24.

In Table 3 through 8, equations (82) and (83) are estimated from the sample already described.[39] The sample of monthly observations spans the period from July 1969 to June 1976 (6907 to 7606). The estimates are made over the entire sample with a dummy variable included for the period 6907 to 7301 to represent the period of the draft (Tables 3 and 4). The estimates were also made separately on the 6907 to 7301 subsample (Tables 5 and 6) and on the all volunteer period 7302 to 7606 (Tables 7 and 8). The sample periods and form of the dependent variable are identified in each table. The SP, SU and AU dummy variables, as identified in Section VI, represent seasonal periods spring, summer and autumn. Two forms of the dependent variable were used in the logarithmic equations estimated. The log of the enlistment rate LNERTE was used as the dependent variable in (82) and (83). The other form chosen was the log of $\frac{R}{1-R}$ where R is the enlistment rate. The latter form is referred to as the LOGIT model (see Cooper, 1977; Theil, 1971).

The results for the entire sample are mixed as might be expected for such a heterogeneous period. The equations have rather high R^2 values, but the Durbin-Watson statistic (D–W) indicates the presence of a good deal of serial correlation.

The DFT dummy is not significant and does not permit the periods to be differentiated by the intercepts of the reduced form equation. There is no persistent pattern to the seasonal dummies.

The coefficient of the wage ratio (W^R) is negative at high levels of significance with both forms of dependent variables. This coefficient is $\epsilon^{\alpha}_{W_r} + \pi_r$ where $\epsilon^{\alpha}_{W_r} > 0$, and $\pi_r < 0$. Thus the negative coefficient suggests that a higher military/civilian wage ratio induces improved retention, thus lowering the observed accession rate. When each wage enters the regression separately the respective coefficients are negative and significant.[40]

This pattern does not hold up over all subsamples, however, and it is difficult to attach any significance to this finding. The pattern for the employment and force level variables is also unstable for this sample.

The somewhat more homogeneous sample for the period 6907 to 7301, during which the draft was in effect yields more interesting results. The D–W statistic indicates that, while some serial correlation remains, it is less of a problem. For this sample, the wage ratio coefficient is negative at very high levels of significance. A clearer pattern emerges for the wages entered separately: the military wage has a negative

[39]The actual regressions contain two variables not discussed in the theoretical discussion above. These are the number of personnel authorized in paygrade E1 (denoted as E1 in the regressions) and the labor force participation rate (denoted as LFPR). The first of these is used as a proxy for the desired maturity of the force. The second is an alternative to the employment rate.

[40]That is, both increases in civilian and increases in military wages reduce net enlistments. In a properly specified model if the wage ratio were the relevant variable the coefficients of W_m and W_c, should have been equal but opposite in sign. However for reasons which will be elucidated below some of the models estimated are not properly specified.

Table 3. **OLSQ Regression Estimate of Reduced Form Model: Accessions by Paydate = TAFMSD**

		Coefficients of Logs of Independent Variables							Dummy Variables					
Period	Dep. Var.	W^R	W^m	W^c	EMP	FORL	E1	LFPR	SP	SU	AU	DFT	R^2	D-W
6907-7606	LNERTE	−1.03498			0.12748	2.62207			0.05385	0.12781	0.04794	0.28076	0.786	0.704
		(4.490)			(0.068)	(2.228)			(0.411)	(0.856)	(0.333)	(1.552)		
		−1.12817			0.23827	2.53804	−0.44220		0.03088	0.11597	1.04530	0.07122	0.797	0.770
		(−4.898)			(0.130)	(2.200)	(−2.051)		(0.240)	(0.792)	(0.322)	(0.348)		
		−1.07964				2.70373	−0.43260	−0.66548	0.04909	0.33854	0.08013	0.03473	0.798	0.750
		(−4.637)				(3.328)	(−2.001)	(−0.436)	(0.387)	(0.671)	(0.583)	(0.175)		
			−0.84371	−3.12012	−0.42281				0.09531	0.13256	0.08742	0.01810	0.812	0.732
			(−3.972)	(−2.788)	(−0.282)				(0.775)	(1.030)	(0.675)	(0.101)		
			−1.00826	−14.63520	4.99725	−13.91930			0.11071	−0.24054	−0.11668	−0.05808	0.856	0.937
			(−5.308)	(−5.665)	(2.887)	(−4.821)			(1.023)	(−1.755)	(−0.960)	(−0.363)		
			−1.02517	−14.24680	4.90097	−13.54120	−0.07723		0.10535	−0.23387	−0.11323	−0.08663	0.857	0.945
			(−5.239)	(−5.135)	(2.788)	(−4.433)	(−0.399)		(0.961)	(−1.684)	(−0.924)	(−0.494)		
			−0.81528	−3.03523				−0.84458	0.10187	0.38730	0.10511	−0.00042	0.813	0.720
			(−3.732)	(−3.230)				(−0.585)	(0.845)	(0.817)	(0.805)	(−0.002)		
			−0.89447	−10.76700		−8.03309	−0.15282	0.19452	0.16128	−0.05969	0.05673	−0.20135	0.842	0.872
			(−4.236)	(−4.122)		(−3.254)	(−0.757)	(0.142)	(1.396)	(−0.130)	(0.464)	(−1.092)		

Table 4. **OLSQ Regression Estimates of Reduced Form Model: Accessions by Paydate = TAFMSD**

		Coefficients of Logs of Independent Variables							Dummy Variables					
Period	Dep. Var.	W^R	W^m	W^c	EMP	FORL	E1	LFPR	SP	SU	AU	DFT	R^2	D-W
6907-7606	LOGIT	−1.03561 (−4.491)			0.12680 (0.068)	2.62217 (2.228)			0.05390 (0.412)	0.12787 (0.856)	0.04797 (0.333)	0.28090 (1.552)	0.786	0.704
		−1.12882 (−4.899)			0.23763 (0.130)	2.53811 (2.200)	−0.44235 (−2.051)		0.03092 (0.240)	0.11602 (0.792)	0.04534 (0.322)	0.07129 (0.348)	0.797	0.770
		−1.40454 (−6.834)						−0.12208 (−0.075)	0.09069 (0.664)	0.08007 (0.150)	0.00305 (0.021)	0.61912 (4.387)	0.756	0.678
		−1.08030 (−4.638)				2.70355 (3.327)	−0.43274 (−2.001)	−0.66594 (−0.436)	0.04912 (0.387)	0.33871 (0.671)	0.08015 (0.583)	0.03481 (0.175)	0.798	0.750
			−1.00888 (−5.310)	−14.64060 (−5.666)	4.99846 (2.886)	−13.92560 (−4.822)			0.11078 (1.023)	−0.24063 (−1.755)	−0.11671 (−0.960)	−0.05808 (−0.365)	0.857	0.937
			−1.02579 (−5.241)	−14.25220 (−5.135)	4.90218 (2.788)	−13.54750 (−4.434)	−0.07723 (−0.398)		0.10542 (0.961)	−0.23395 (−1.684)	−0.11326 (−0.924)	−0.08663 (−0.494)	0.857	0.945
			−0.89505 (−4.237)	−10.77150 (−4.122)		−8.03798 (−3.256)	−0.15284 (−0.756)	0.19444 (0.142)	0.16136 (1.396)	−0.05969 (−0.130)	0.05675 (0.464)	−0.20138 (−1.091)	0.842	0.872
		−0.12288 (−1.103)				−0.60873 (−1.441)			−0.00097 (−0.015)	0.16837 (2.532)	0.12175 (1.869)	0.28517 (3.423)	0.360	0.997
		−0.19751 (−1.861)				−0.63725 (−1.617)	−0.37324 (−3.526)		−0.01846 (−0.305)	0.16215 (2.613)	0.12298 (2.023)	0.10421 (1.119)	0.450	1.216

Table 5. **OLSQ Regression Estimates of Reduced Form Model: Accessions by Paydate = TAFMSD**

		Coefficients of Logs of Independent Variables							Dummy Variables					
Period	Dep. Var.	w^R	w^m	w^c	EMP	FORL	E1	LFPR	SP	SU	AU	DFT	R^2	D-W
6907-7301	LNERTE	−1.54926			−4.62270				0.04147	0.06838	0.16464		0.715	1.364
		(−9.439)			(−2.552)				(0.255)	(0.440)	(1.080)			
		−1.96770				−3.54802			−0.03964	−0.17498	−0.03392		0.730	1.357
		(−8.408)				(−2.995)			(−0.259)	(−1.132)	(−0.235)			
		−1.88549		−2.29528	−2.63102				0.00674	−0.08905	0.04247		0.738	1.448
		(−7.649)		(−1.049)	(−1.788)				(0.042)	(−0.510)	(0.260)			
		−1.99569				−4.19391	0.34456		−0.05050	−0.17599	−0.03428		0.737	1.357
		(−8.460)				(−3.093)	(0.980)		(−0.329)	(−1.138)	(−0.235)			
		−1.91966			−2.00847	−3.29720	).29426		−0.00833	−0.10065	0.03261		0.743	1.433
		(−7.646)			(−0.902)	(−2.958)	(0.825)		(−0.572)	(0.199)				
			−1.46133	−9.87041	2.68999	−12.84740	0.46507		0.01427	−0.25415	−0.02916		0.775	1.179
			(−4.613)	(−1.837)	(0.895)	(−2.773)	(1.338)		(0.093)	(−1.402)	(−0.184)			
			−1.48860	−5.35690			−7.49935						0.745	1.273
			(−5.282)	(−1.476)			(−3.092)							
			−1.50854	−5.59752			−8.41587	0.38765					0.754	1.261
			(−5.368)	(−1.547)			(−3.317)	(1.172)						

Table 6. **OLSQ Regression Estimates of Reduced Form Model: Accessions by Paydate = TAFMSD**

Period	Dep. Var.	Coefficients of Logs of Independent Variables: W^R	W^m	W^c	EMP	FORL	E1	LFPR	Dummy Variables: SP	SU	AU	DFT	R^2	D-W
6907-7301	LOGIT	−1.96871				−3.55060			−0.3960	−0.17501	−0.03394		0.731	1.356
		(−8.409)				(−2.996)			(−0.259)	(−1.132)	(−0.232)			
		−1.99672				−4.19689	0.34477		−0.05046	−0.17603	−0.03431			
		(−8.462)				(−3.094)	(0.980)		(−0.329)	(−1.138)	(−0.235)			
		−1.92062			−2.01037	−3.29933	0.29442		−0.00826	−0.10062	0.03265		0.744	1.433
		(−7.647)			(−0.903)	(−1.958)	(0.825)		(−0.051)	(−0.571)	(0.199)			
		−1.54997			−4.62631				0.04157	0.06852	0.16477		0.715	1.364
		(−9.440)			(−2.553)				(0.256)	(0.441)	(1.081)			
		−1.88643			−2.29734	−2.63278			0.00682	−0.08901	0.04252		0.739	1.448
		(−7.650)			(−1.049)	(−1.789)			(0.043)	(−0.509)	(0.261)			
			−1.82179	3.68943	−4.03725				0.01647	0.00235	0.10388		0.724	1.453
			(−5.971)	(1.816)	(−2.132)				(0.100)	(0.014)	(0.638)			
			−1.81990	4.85576				−1.92927	−0.04670	0.53599	0.05069		0.697	1.136
			(−5.554)	(2.370)				(−0.953)	(−0.276)	(0.764)	(0.290)			
			−1.46212	−9.87370	2.68978	−12.85300	0.46530		0.01435	−0.25418	−0.02914		0.775	1.179
			(−4.614)	(−1.837)	(0.894)	(−2.773)	(1.338)		(0.094)	(−1.402)	(−0.184)			

Table 7. **OLSQ Regression Estimates of Reduced Form Model: Accessions by Paydate = TAFMSD**

		Coefficients of Logs of Independent Variables							Dummy Variables					
Period	Dep. Var.	W^R	W^m	W^c	EMP	FORL	E1	LFPR	SP	SU	AU	DFT	R^2	D-W
7302-7606	LNERTE	−2.73747 (−0.957)				6.28489 (6.683)	0.01710 (0.077)		0.09330 (0.732)	0.19766 (1.206)	−0.04741 (−0.339)		0.677	1.640
		−2.71184 (−0.968)				6.26015 (7.184)			0.09201 (0.738)	0.19912 (1.241)	−0.04595 (−0.336)		0.677	1.635
		−2.38779 (−0.780)			0.58293 (0.282)	5.91500 (3.922)			0.08412 (0.650)	0.18695 (1.111)	−0.06369 (−0.419)		0.677	1.661
		−2.37688 (−0.743)			0.59327 (0.268)	5.90390 (3.448)	−0.00344 (−0.014)		0.08373 (0.624)	0.18703 (1.094)	−0.06371 (−0.413)		0.677	1.660
			−1.86515 (−0.561)	4.30428 (0.995)		8.67139 (1.724)			0.07788 (0.602)	0.26896 (1.242)	−0.03023 (−0.213)		0.679	1.702
			−1.86683 (−0.553)	4.35999 (0.736)	−0.03678 (−0.014)	8.74658 (1.181)			0.07806 (0.592)	0.27128 (0.986)	−0.02876 (−0.162)		0.679	1.702
			−1.82994 (−0.518)	4.34347 (0.720)	−0.01090 (−0.004)	8.73649 (1.161)	−0.01030 (−0.043)		0.07682 (0.560)	0.27220 (0.972)	−0.02853 (−0.158)		0.679	1.701

Table 8. **OLSQ Regression Estimates of Reduced Form Model: Accessions by Paydate = TAFMSD**

		Coefficients of Logs of Independent Variables							Dummy Variables					
Period	Dep. Var.	W^R	W^m	W^c	EMP	FORL	E1	LFPR	SP	SU	AU	DFT	R^2	D-W
7302-7606	LOGIT	−2.73797				6.28590	0.01707		0.09330	0.19769	−0.04743		0.677	1.640
		(−0.957)				(6.683)	(0.077)		(0.732)	(1.206)	(−0.339)			
		−2.38824			0.58307	5.91596			0.08413	0.18698	−0.06371		0.677	1.661
		(−0.780)			(0.282)	(3.922)			(0.650)	(1.111)	(−0.419)			
		−2.71237				6.26119			0.09202	0.19915	−0.04596		0.677	1.635
		(−0.968)				(7.185)			(0.738)	(1.241)	(−0.336)			
			−1.86719	4.36082	−0.03675	8.74808			0.07807	0.27132	−0.02877		0.679	1.702
			(−0.553)	(0.736)	(−0.014)	(1.181)			(0.592)	(0.986)	(−0.162)			
			−4.79844	3.58631		5.37835	−0.00887						0.646	1.568
			(−2.128)	(0.906)		(1.573)	(−0.041)							
			−4.80442	3.60889		5.40922							0.646	1.569
			(−2.160)	(0.923)		(1.612)								
		−2.37723			0.59350	5.90477	−0.00347		0.08373	0.18706	−0.06373		0.677	1.660
		(−0.743)			(0.268)	(3.448)	(−0.015)		(0.624)	(1.094)	(−0.413)			

and highly significant relation to accessions, and the civilian wage fades to low significance. This suggests that, even during the period of the draft when the supply to the Air Force was encouraged, the primary effect of the Air Force wage was to reduce accessions because of a strong retention effect. The pattern of regression coefficient sizes for the employment variable shows great sensitivity to the variables included with it in the regression. Moreover, this period is characterized by observations on the demand side of the market, so what is revealed in the employment coefficient is the relation of Air Force demand to other variables that are correlated with $(1 - U)$. This observation is further supported by the negative coefficient on the force level variable. In a non-draft period this coefficient would be positive for a system in steady state. The negative coefficient appears to be due to the large force level changes that occurred over the period and the presence in the force of large numbers of individuals with common enlistment periods. When force level requirements were reduced late in the period, a large proportion of the force also ended its tour thus requiring high levels of accession flows.

The most stable and interesting results are those covering the volunteer period 7302 to 7606. The D–W statistic indicates a reasonable lack of serial correlation in the sample observations. The wage ratio coefficient is negative, but statistically insignificant in the LNERTE regressions (Table 7). The force level variable dominates all other variables in terms of significance, which is to be expected since the reductions in force level that took place over this period were by far the major changes in the underlying exogenous variables.[41]

In Table 8 the coefficient of the wage ratio is negative but not significantly so. When the wages are entered separately, the coefficient of the military wage becomes significantly negative, while the civilian wage coefficient is positive but not significant.[42] Both these coefficient signs suggest that the equilibrium accession rate is demand-determined. A rise in the military wage reduces turnover and accession demand, thus lowering observed accessions. A higher civilian wage increases turnover and thus accession demand and so leads to an increase in observed accessions. The positive but insignificant coefficient of employment opportunities is also consistent with this view. Enhanced civilian employment opportunities reduce retention, thereby increasing accession demand leading to a higher observed accession rate.

Accessions: Paydate

An enlistee's paydate marks the time he has executed an enlistment contract and begins accruing longevity but has not begun active duty. Since paydate records the time an individual enlists, the number of accessions by paydate is the closest counterpart to supply available in the sample. Some enlistees will begin service on their paydate and others will begin later, however, this timing would seem to depend more on the personnel management practices of the Air Force, and particularly on the need to smooth out the random fluctuations in enlistments to meet force level requirements and manage training school loads.

Estimates of the reduced form equation (83) are presented in Tables 9 through 14. The estimates for the entire sample do accord somewhat more importance to the draft dummy than those obtained for paydate = service date accessions. The pattern of seasonal dummy variables also suggests that the supply of enlistments is heaviest in the spring, whereas accessions by service date were heaviest in the summer, reflecting the carry-over of the heavy spring enlistment surge. However, for the all volunteer period 7302 – 7606 the enlistment surge occurs in the winter quarter (see Tables 13 and 14). This is consistent with the view that during the volunteer period fewer enlistees are drawn from college, quite possible because, in the absence of the draft, the inducement to attend college is lessened.

Throughout the estimated equations the coefficient of the wage ratio is positive but insignificant. For the equations estimated by paydate = service date (Table 3 through 8) and by service date (Tables 15 through 20) this coefficient is consistently negative. This does support the view that enlistments by paydate

[41] These results point up the importance of the retention market in the determinations of accessi[illegible]

[42] In this case the signs of the two wages are as predicted opposite and moreover in two of th[illegible] ssions they are not significantly different in absolute value indicating that for this sample period and definition of accessions the relative wage variable is relevant.

Table 9. **OLSQ Regression Estimates of Reduced Form Model: Accessions by Paydate**

		Coefficients of Logs of Independent Variables							Dummy Variables					
Period	Dep. Var.	W^R	W^m	W^c	EMP	FORL	E1	LFPR	SP	SU	AU	DFT	R^2	D-W
6907-7606	LNERTE	0.06175			−1.26903	0.47750			0.08632	0.13532	0.00538	0.2371	0.329	0.701
		(0.506)			(−1.287)	(0.767)			(1.246)	(1.712)	(0.071)	(2.430)		
		0.01072			−1.20836	0.43149	−0.24214		0.07374	0.12883	0.00393	0.11796	0.367	0.737
		(0.088)			(−1.253)	(0.708)	(−2.127)		(1.085)	(1.666)	(0.053)	(1.092)		
		−0.02833				−0.09590	−0.24318	−0.24128	0.05394	0.15714	−0.03119	0.15919	0.355	0.758
		(−0.228)				(−0.221)	(−2.107)	(−0.296)	(0.797)	(0.583)	(−0.425)	(1.499)		
			0.07615	−8.50829	1.35589	−8.43865			0.11697	−0.06323	−0.08336	0.05006	0.559	0.974
			(0.765)	(−6.281)	(1.494)	(−5.574)			(2.061)	(−0.880)	(−1.308)	(0.600)		
			0.06610	8.27746	1.29866	−8.21392	−0.04591		0.11378	−0.05926	−0.08131	0.03309	0.560	0.974
			(0.691)	(6.633)	(1.409)	(−5.130)	(−0.452)		(1.979)	(−.0814)	(−1.266)	(0.360)		

Table 10. **OLSQ Regression Estimates of Reduced Form Model: Accessions by Paydate**

Period	Dep. Var.	Coefficients of Logs of Independent Variables: W^R	W^m	W^c	EMP	FORL	E1	LFPR	Dummy Variables: SP	SU	AU	DFT	R^2	D-W
6907-7606	LOGIT	0.06173			−1.27013	0.47766			0.08640	0.13545	0.00539	0.23294	0.329	0.701
		(0.506)			(−1.287)	(0.766)			(1.246)	(1.713)	(0.071)	(2.430)		
		(0.10662			−1.20941	0.43161	−0.24235		0.07381	0.12896	0.003944	0.11809	0.367	0.737
		(0.876)			(−1.253)	(0.708)	(−2.127)		(1.085)	(1.667)	(0.05294)	(1.092)		
		−0.02842				−0.09624	−0.24339	−0.24158	0.05401	0.15731	−0.03121	0.15936	0.355	0.759
		(−0.228)				(−0.222)	(−2.107)	(−0.296)	(0.797)	(0.538)	(−0.425)	(1.499)		
			0.07614	−8.51536	1.35698	−8.44596			0.11708	−0.06327	−0.08342	0.05014	0.559	0.974
			(0.764)	(−6.281)	(1.494)	(−5.574)			(2.061)	(−0.879)	(−1.308)	(0.6003)		
			0.06608	−8.28430	1.29970	−8.22101	−0.04595		0.11389	−0.059299	−0.081368	0.03315	0.560	0.974
			(0.644)	(−5.691)	(1.409)	(−5.130)	(−0.452)		(1.979)	(−0.814)	(−1.266)	(0.360)		
			0.08788	−7.41211		−6.83974	−0.06767	0.29856	0.12447	−0.09280	−0.45902	0.01108	0.549	0.948
			(0.824)	(−5.617)		(−5.485)	(−0.663)	(0.431)	(2.133)	−0.40131	(−0.734)	(0.119)		

Table 11. **OLSQ Regression Estimates of Reduced Form Model: Accessions by Paydate**

Period	Dep. Var.	Coefficients of Logs of Independent Variables							Dummy Variables					
		W^R	W^m	W^c	EMP	FORL	E1	LFPR	SP	SU	AU	DFT	R^2	D-W
6907-7301	LNERTE	−0.37222			−2.31469	−2.39442			0.21001	0.22363	0.07046		0.636	1.415
		(−3.349)			(−2.345)	(−3.610)			(2.928)	(2.838)	(0.958)			
		−0.38433			−2.21300	−2.63062	0.10433		0.20467	0.21952	0.06697		0.640	1.434
		(−3.382)			(−2.196)	(−3.451)	(0.646)		(2.812)	(2.754)	(0.901)			
			−0.29581	−1.89274	−1.30557	−4.47510	0.13732		0.20904	0.18987	0.05504		0.648	1.377
			(−1.953)	(−0.737)	(−0.908)	(−2.020)	(0.826)		(2.857)	(2.191)	(0.726)			
		−0.45512				−3.31917			0.16325	0.13698	−0.00657		0.580	1.381
		(−4.078)				(−5.876)			(2.237)	(1.858)	(−0.094)			
		−0.46810				−3.61865	0.15976		0.15821	0.13651	−0.00674		0.590	1.423
		(−4.158)				(−5.591)	(0.952)		(2.160)	(1.850)	(−0.097)			

Table 12. **OLSQ Regression Estimates of Reduced Form Model: Accessions by Paydate**

		Coefficients of Logs of Independent Variables							Dummy Variables					
Period	Dep. Var.	W^R	W^m	W^c	EMP	FORL	E1	LFPR	SP	SU	AU	DFT	R^2	D-W
6907-7301	LOGIT	−0.45560				−3.32238			0.16340	0.13711	−0.00657		0.580	1.381
		(−4.078)				(−5.876)			(2.237)	(1.858)	(−0.094)			
		−0.46859				−3.62221	0.15995		0.15836	0.13664	−0.00674		0.590	1.423
		(−4.159)				(−5.592)	(0.952)		(2.160)	(1.850)	(−0.097)			
		−0.37261			−2.31691	−2.39674			0.21021	0.22385	0.07054		0.636	1.415
		(−3.349)			(−2.345)	(−3.610)			(2.928)	(2.838)	(0.958)			
		−0.38474			−2.21509	−2.63326	0.10447		0.20486	0.21973	0.0670		0.640	1.434
		(−3.383)			(−2.197)	(−3.452)	(0.646)		(2.812)	(2.754)	(0.901)			
			−0.29618	−1.89349	−1.30720	−4.47867	0.13748		0.20923	0.19006	0.05510		0.648	1.377
			(−1.954)	(−0.736)	(−0.908)	(−2.020)	(0.826)		(2.857)	(2.192)	(0.727)			

Table 13. **OLSQ Regression Estimates of Reduced Form Model: Accessions by Paydate**

		Coefficients of Logs of Independent Variables							Dummy Variables					
Period	Dep. Var.	w^R	w^m	w^c	EMP	FORL	E1	LFPR	SP	SU	AU	DFT	R^2	D-W
7302-7606	LNERTE	0.86253				1.51921	−0.00608		−0.05322	−0.04802	−0.14078		0.428	1.458
		(0.549)				(2.941)	(−0.050)		(−0.760)	(−0.533)	(−1.832)			
		0.85342				1.52800			−0.05276	−0.04854	−0.14130		0.428	1.459
		(0.555)				(3.193)			(−0.771)	(−0.551)	(−1.884)			
		0.28223			−1.02751	2.13637			−0.03886	−0.02709	−0.11003		0.441	1.406
		(0.170)			(−0.916)	(2.607)			(−0.553)	(−0.296)	(−1.332)			
		0.17791			−1.12641	2.24256	0.03291		−0.03504	−0.02784	−0.10984		0.442	1.413
		(0.102)			(−0.937)	(2.413)	(0.255)		(−0.481)	(−0.300)	(−1.312)			
			−1.58032	−5.43034		−5.40254			−0.01214	−0.24928	−0.18648		0.536	1.461
			(−0.958)	(−2.529)		(−2.164)			(−0.189)	(−2.319)	(−2.650)			
			−1.52877	−7.13740	1.12655	−7.70615			−0.01780	−0.32020	−0.23144		0.547	1.520
			(−0.932)	(−2.456)	(0.875)	(−2.121)			(−0.275)	(−2.373)	(−2.650)			
			−1.73258	−7.04609	0.98359	−7.65042	0.05688		−0.01093	−0.32530	−0.23269		0.550	1.531
			(−1.003)	(−2.391)	(0.736)	(−2.080)	(0.482)		(−0.163)	(−2.376)	(−2.632)			

Table 14. **OLSQ Regression Estimates of Reduced Form Model: Accessions by Paydate**

		Coefficients of Logs of Independent Variables							Dummy Variables					
Period	Dep. Var.	W^R	W^m	W^c	EMP	FORL	E1	LFPR	SP	SU	AU	DFT	R^2	D-W
7302-7606	LOGIT	0.86323				1.52029	-0.00610		-0.05326	-0.04806	-0.14089		0.428	1.458
		(0.549)				(2.941)	(-0.050)		(-0.760)	(-0.533)	(-1.833)			
		0.85409				1.52911			-0.05280	-0.04858	-0.14141		0.428	1.459
		(0.555)				(3.193)			(0.0771)	(-0.551)	(-1.884)			
		0.17819			-1.12709	2.24407	0.03292		-0.03507	-0.02786	-0.10993		0.442	1.413
		(0.103)			(-0.937)	(2.413)	(0.255)		(-0.481)	(-0.300)	(-1.312)			
		0.28254			-1.02817	2.13787			-0.03888	-0.02711	-0.11012		0.441	1.406
		(0.170)			(-0.916)	(2.607)			(-0.553)	(-0.291)	(-1.332)			
			-1.52966	-7.14221	1.12730	-7.71113			-0.01781	-0.32042	-0.23161		0.547	1.520
			(-0.923)	(-2.456)	(0.857)	(-2.120)			(-0.275)	(-2.373)	(-2.650)			

Table 15. **OLSQ Regression Estimates of Reduced Form Model: Accessions by TAFMSD**

		Coefficients of Logs of Independent Variables							Dummy Variables					
Period	Dep. Var.	W^R	W^m	W^c	EMP	FORL	E1	LFPR	SP	SU	AU	DFT	R^2	D-W
6907-7606	LNERTE	−0.12138			−0.03162	−0.59365			−0.00034	0.16951	0.12282	0.28351	0.360	0.995
		(−1.015)			(−0.033)	(−0.973)			(−0.005)	(2.189)	(1.646)	(3.021)		
		−0.20283			−0.29567	−0.29359	−0.42928		−0.02122	0.17642	0.13632		0.442	1.205
		(−1.782)			(−0.353)	(−0.654)	(−4.649)		(−0.334)	(2.500)	(2.003)			
		−0.20000			0.06187	−0.66456	−0.37314		−0.01972	0.15951	0.12060	0.10670	0.450	1.221
		(−1.758)			(0.069)	(−1.166)	(−3.504)		(−0.310)	(2.206)	(1.732)	(1.056)		
		−0.14092				−0.55130	−0.35994	−0.98460	0.00129	0.47705	0.16140	0.06494	0.462	1.175
		(−1.238)				(−1.388)	(−3.405)	(−1.318)	(0.021)	(1.932)	(2.403)	(0.668)		
			−0.10962	−6.77374	2.11116	−7.87212			0.02468	0.00742	0.05038	0.13443	0.512	1.277
			(−1.043)	(−4.740)	(2.204)	(−4.928)			(0.412)	(0.098)	(0.749)	(1.527)		
			−0.16123	−5.58840	1.81729	−6.71810	−0.23573		0.0832	0.02781	0.06091	0.04727	0.544	1.406
			(−1.539)	(−3.761)	(1.931)	(−4.107)	(−2.272)		(0.142)	(0.374)	(0.928)	(0.503)		
			−0.07393	−4.14518		−4.43588	−0.25872	−067345	0.04188	0.33297	0.15293	−0.02048	0.527	1.252
			(−0.675)	(−3.057)		(−3.46257)	(−2.468)	(−0.945)	(0.698)	(1.402)	(2.409)	(−0.214)		

Table 16. **OLSQ Regression Estimates of Reduced Form Model: Accessions by TAFMSD**

		Coefficients of Logs of Independent Variables							Dummy Variables					
Period	Dep. Var.	W^R	W^m	W^c	EMP	FORL	E1	LFPR	SP	SU	AU	DFT	R^2	D-W
6907-7606	LOGIT	−0.12153			−0.03149	−0.59445			−0.00033	0.16964	0.12291	0.28379	0.360	0.995
		(−1.016)			(−0.033)	(−0.973)			(−0.005)	(2.189)	(1.646)	(3.022)		
		−0.20022			(0.069)	(−1.167)	(−3.504)		(−0.310)	(2.206)	(1.732)	(1.056)		
		−0.14108				−0.55200	−0.36025	−0.98542	0.00130	0.47745	0.16153	0.06503	0.462	1.175
		(−1.238)				(−1.388)	(−3.405)	(−1.318)	(0.021)	(1.932)	(2.403)	(0.668)		
			−0.10976	−6.77968	2.11318	−7.87934			0.02471	0.00742	0.05041	0.13457	0.512	1.277
			(−1.044)	(−4.740)	(2.204)	(−4.929)			(0.412)	(0.098)	(0.749)	(1.527)		
			−0.16141	−5.59332	1.81906	−6.72435	−0.23593		0.00833	0.02782	0.06095	0.04734	0.544	1.406
			(−1.539)	(−3.761)	(1.931)	(−4.108)	(−2.272)		(0.142)	(0.374)	(0.928)	(0.504)		
			−0.07403	−4.14872		−4.43993	−0.25894	−0.67400	0.04192	0.33325	0.15306	−0.02046	0.527	1.242
			(−0.675)	(−3.057)		(−3.463)	(−2.468)	(−0.945)	(0.699)	(1.401)	(2.409)	(−0.214)		

Table 17. **OLSQ Regression Estimates of Reduced Form Model: Accessions by TAFMSD**

		Coefficients of Logs of Independent Variables							Dummy Variables					
Period	Dep. Var.	W^R	W^m	W^c	EMP	FORL	E1	LFPR	SP	SU	AU	DFT	R^2	D-W
6907-7301	LNERTE	−0.65324 (−6.127)			0.46900 (0.495)	−4.21623 (−6.627)			0.06158 (0.895)	0.01777 (0.235)	0.06663 (0.945)		0.688	1.619
		−0.63644 (−6.361)				−4.02886 (−7.956)			0.07105 (1.086)	0.03533 (0.535)	0.08224 (1.317)		0.686	1.581
		−0.61808 (−6.243)				−3.60535 (−6.335)	−0.22593 (−1.531)		0.07817 (1.214)	0.03599 (0.555)	0.08248 (1.344)		0.705	1.763
		−0.62774 (−5.895)			0.25502 (0.270)	−3.71921 (−5.207)	−0.21954 (−1.451)		0.07282 (1.068)	0.02643 (0.354)	0.07399 (1.062)		0.705	1.777
			−0.69682 (−4.892)	2.40482 (0.995)	−0.45318 (−0.335)	−2.27973 (−1.094)	−0.24529 (−.569)		0.06941 (1.009)	0.04956 (0.608)	0.08330 (1.691)		0.710	1.900

Table 18. **OLSQ Regression Estimates of Reduced Form Model: Accessions by TAFMSD**

Period	Dep. Var.	Coefficients of Logs of Independent Variables							Dummy Variables					
		W^R	W^m	W^c	EMP	FORL	E1	LFPR	SP	SU	AU	DFT	R^2	D-W
6907-7301	LOGIT	−0.63706 (−6.362)				−4.03277 (−7.957)			0.07113 (1.087)	0.03537 (0.535)	0.08232 (1.317)		0.686	1.581
		−0.61869 (−6.243)				−3.60897 (−6.335)	−0.22608 (−1.531)		0.07825 (1.214)	0.03603 (0.555)	0.08256 (1.344)		0.705	1.763
		−0.62835 ()5.895)			0.25519 (0.270)	−3.72291 (−5.207)	−0.21969 (−1.450)		0.07289 (1.068)	0.02646 (0.354)	0.07406 (1.062)		0.705	1.777
		−0.65387 (−6.127)			0.46932 (0.495)	−4.22028 (−6.627)			0.06164 (0.895)	0.01780 (0.235)	0.06670 (0.945)		0.688	1.619
			−0.69755 (−4.892)	2.40850 (0.996)	−0.45422 (−0.336)	−2.28095 (−1.094)	−0.24548 (−1.569)		0.06948 (1.009)	0.04964 (0.609)	0.08339 (1.169)		0.710	1.900

Table 19. **OLSQ Regression Estimates of Reduced Form Model: Accessions by TAFMSD**

Period	Dep. Var.	Coefficients of Logs of Independent Variables							Dummy Variables					
		W^R	W^m	W^c	EMP	FORL	E1	LFPR	SP	SU	AU	DFT	R^2	D-W
7302-7606	LNERTE	−0.20272				1.21841			−0.7862	0.19508	0.07941		0.338	2.074
		(−0.017)				(2.267)			(−1.023)	(1.971)	(0.943)			
		−1.26228			−1.90609	2.34695			−0.05282	0.23488	0.13743		0.381	2.025
		(−0.691)			(−1.546)	(2.607)			(−0.648)	(2.339)	(1.515)			
		−1.02117			−1.67748	2.10154	−0.07607		−0.06164	0.23660	0.13699		0.387	2.023
		(−0.537)			(−1.275)	(2.065)	(−0.539)		(−0.773)	(2.330)	(1.494)			
			−1.47745	−2.19461		−2.41169			−0.05734	0.08994	0.05575		0.365	1.959
			(−0.734)	(−0.837)		(−0.791)			(−0.732)	(0.685)	(0.649)			
			−1.54907	0.17685	−1.56500	0.78849			−0.04949	0.18846	0.11821		0.384	1.995
			(−0.769)	(0.050)	(−0.999)	(0.178)			(−0.628)	(1.477)	(1.112)			
			−1.28853	0.06013	−1.38225	0.71724	−0.07271		−0.05826	0.19498	0.19980		0.388	1.997
			(−0.613)	(0.017)	(−0.850)	(0.160)	(−0.506)		(−0.715)	(1.170)	(1.114)			

Table 20. **OLSQ Regression Estimates of Reduced Form Model: Accessions by TAFMSD**

Period	Dep. Var.	Coefficients of Logs of Independent Variables: w^R	w^m	w^c	EMP	FORL	E1	LFPR	Dummy Variables: SP	SU	AU	DFT	R^2	D-W
7302-7606	LOGIT	−0.20300				1.21938			−0.07869	0.19522	0.07946		0.338	2.074
		(−0.117)				(2.267)			(−1.023)	(1.971)	(0.943)			
		−0.00175				1.02517	−0.13422		−0.08878	0.20669	0.09096		0.356	2.087
		(−0.001)				(1.791)	(−0.994)		(−1.144)	(2.072)	(1.069)			
		−1.26324			−1.90730	2.34866			−0.05288	0.23505	0.13751		0.381	2.025
		(−0.691)			(−1.546)	(2.607)			(−0.684)	(2.339)	(1.515)			
		−1.02198			−1.67858	2.10311	−0.07611		−0.06169	0.23678	0.13707		0.387	2.023
		(−0.537)			(−1.275)	(2.066)	(−0.539)		(−0.773)	(2.330)	(1.494)			
			−1.55027	0.17685	−1.56593	0.78882			−0.04954	0.18859	0.11827		0.384	1.995
			(−0.769)	(0.050)	(−0.998)	(0.178)			(−0.629)	(1.478)	(1.112)			

are more indicative of supply activity. It must be kept in mind, however, that the equation estimated is not a supply equation but is a reduced form equation showing the net effect of demand and supply variables on enlistments.

When the military and civilian wages are separately entered into the regression equation for the all volunteer period, the military wage has an insignificant negative sign. This reflects the opposing forces so often referred to in this report. A higher military wage increases retention thus reducing accession demand but also increases accession supply. As a result, the actual accession rate moves slightly downward. The coefficient underestimates the elasticity of retention with respect to the military wage. On the other hand the coefficient of the civilian wage is significantly negative for the volunteer period, but this coefficient is insignificant in the paydate = service date and the service date regressions. This is further evidence that paydate accessions reflect supply activity better than the other variables which are subject to Air Force personnel management policies to a greater extent. Also both the civilian wage and the employment variable have negative signs in contradistrinction to the other regressions which further supports this view. However, the coefficient of W_c cannot be taken to be a supply elasticity with respect to the civilian wage for several reasons. First, the force level coefficient is negative indicating that the estimates of the reduced form equation are probably misspecified. Moreover, the data are plagued by a strong negative correlation ($r = -.98$) between the civilian wage and the force level. Largely, this is due to the fact that the wage rates are not defleated by a price index. While these problems do not affect interpretation of the wage ratio, they do vitiate the results for those equations in which both wages enter separately.

Accessions: Service Date (TAFMSD)

New accessions begin service on their TAFMSD. This date may match the paydate for many individuals but it is later than the paydate for the largest proportion of inductees. There are two reasons why service date is later than paydate. The volunteer may wish to sign up now in order to pick the date of entry into the service. Or the Air Force may wish to carry over personnel from a given month to a future month as a means of smoothing the flow of accessions. In either case it is evident that observed accessions by service date reflect Air Force willingness to allow early sign-ups or to smooth accession flows and thus one would expect accessions by service date to more strongly reflect Air Force demand policies than accessions with paydate equal to service date.

This conjecture is supported by comparing the coefficients of the wage ratio in service date regressions (Tables 15 thru 20) with those in the paydate regressions (Tables 9 through 14). For the volunteer period the wage coefficients for service date accessions are negative (though not significant) while they are positive (but insignificant) for paydate accessions. Note that the equation estimated is still a reduced form, not a supply or demand equation; a reasonable conjecture is simply that variations in accessions by service date are more reflective of variations in demand than supply. This conjecture is further borne out in the regressions for the entire sample. In the paydate regressions the coefficient of the civilian wage is negative but larger in absolute value and statistically more significant than the service date regressions. This finding accords with the notion that the civilian wage, while a factor in Air Force accession demand because of its effect on the retention market, more strongly affects the direct supply of volunteers.

AFQT Distributions

It has been argued throughout this report that if the Air Force wage, and the civilian alternatives of volunteers and the rate at which prospective volunteers search the Air Force are all fixed, then the force level, the quality of personnel, or both will do a random walk. Evidence has been presented (see Appendix D) to support the fact that the personnel inventory does fluctuate, and does not always equal the mandated force level. Some inventory adjustment is made at year-end to meet the year-end force level constraints. Nonetheless, the force level is not permitted to deviate far from the mandated level; monthly personnel inventories match interpolated year-end constraints fairly closely. This raises the issue of what is happening to the AFQT distribution. Does it fluctuate as this theory suggests, and if so does it follow the pattern suggested by the theory?

The theory indicates that in the accession market if the Air Force is to meet its force level constraints, it must be a quality taker. That is, it must take its quantities of monthly accessions from the highest mental category consistent with acquiring the accessions required for that month. If this is so, then whenever supply is plentiful relative to demand, say because the military wage is favorable relative to civilian wage or the civilian market exhibits high unemployment, the AFQT distributions of entering recruits should shift toward higher mental groups. Moreover, the same effect is operable in the retention market and serves to shift the AFQT distribution of those retained whenever the downward fluctuation in the civilian economy is sufficiently great.

In Appendix E, histograms of the AFQT distributions for less than 1 year of service and 4 years of service are given bi-annually for December 1970 through 1976. The behavior of the left- and right-hand tails of the distributions evidences Air Force skimmings (shrinkage of the right-hand tail) during high unemployment years 1974 and 1976. On the other hand there is some contraction of supply, as shown by the shrinkage of the left-hand tail, during better employment years. This is caused by the higher mental groups opting for civilian employment and the Air Force meeting its quota by taking in individuals with lower scores.

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APPENDIX A: CONTINUATION RATES, MEAN TIME IN THE AIR FORCE AND PROMOTION PROBABILITIES

Table A1 displays continuation rates for the total enlisted force. These are conditional probabilities in the sense that they measure the odds that an individual in his ith year of service will survive (continue) to the i + 1st year of service. The available data were generally for 2-year length of service categories. Thus, if C_j represents the continuation rate, a typical entry in Table A1 gives the probability that enlistees in, for example, their first or second year of service remain in the Air Force through their third or fourth year of service. (The j subscript refers to the terminal year of service.)

Table A2 shows the calculated stopping rates, Sj, for the total enlisted force. $S_j = 1 - C_j$.

Table A1. **Total Enlisted Force: Continuation Rates**

Years of Service		1969		1970		1971		1972		1973		1974	
From	To	June	Dec	June	Dec	June	Dec	June	Dec	June	Dec	June	Dec
<1	1–2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1–2	3–4	0.474	0.594	0.602	0.543	0.513	0.538	0.514	0.551	0.531	0.522	0.512	0.586
3	4	0.204	0.169	0.201	0.297	0.347	0.326	0.349	0.346	0.354	0.370	0.463	0.436
4	5	0.840	0.847	0.851	0.904	0.964	0.950	0.926	0.940	0.921	0.926	0.927	0.906
3–4	4–5	0.306	0.291	0.377	0.522	0.605	0.464	0.422	0.439	0.509	0.580	0.626	0.603
4–5	6–7	0.847	0.856	0.858	0.853	0.879	0.849	0.836	0.846	0.820	0.782	0.771	0.803
6–7	8–9	0.714	0.755	0.818	0.824	0.883	0.805	0.775	0.776	0.776	0.775	0.776	0.777
8–9	10–11	0.925	0.887	0.959	0.964	0.976	0.961	0.943	0.944	0.934	0.917	0.928	0.931
10–11	12–13	0.915	0.932	0.963	0.961	0.963	0.957	0.951	0.949	0.942	0.930	0.954	0.951
12–13	14–15	0.973	0.979	0.997	0.988	0.995	0.989	0.979	0.977	0.972	0.957	0.983	0.985
14–15	16–17	0.990	0.985	1.001	0.987	0.989	0.989	0.985	0.986	0.979	0.963	0.996	0.997
16–17	18–19	0.993	0.989	1.006	0.994	0.991	0.996	0.991	0.989	0.980	0.964	0.967	0.994
18–19	20–21	0.419	0.416	0.412	0.376	0.340	0.347	0.364	0.384	0.400	0.377	0.383	0.438
20–21	22–23	0.439	0.507	0.508	0.519	0.521	0.504	0.479	0.465	0.493	0.464	0.481	0.534
22–23	24–25	0.594	0.632	0.615	0.622	0.575	0.607	0.576	0.553	0.575	0.554	0.588	0.648
24–25	26–27	0.565	0.595	0.612	0.601	0.532	0.500	0.447	0.384	0.370	0.325	0.346	0.433
26–27	28–29	0.602	0.633	0.608	0.584	0.515	0.453	0.357	0.268	0.269	0.229	0.227	0.329
28–29	30–31	0.187	0.180	0.180	0.191	0.145	0.123	0.085	0.041	0.022	0.006	0.008	0.014

Note. — Source: Air Force Systems Command, Air Force Human Resources Laboratory, Computational Sciences Division, 25 July 1977.

Table A2. **Total Enlisted Force: Stopping Rates**

Years of Service		1969		1970		1971		1972		1973		1974		1975	
From	To	June	Dec	June	Dec	June	Dec	June	Dec	June	Dec	June	Dec	June	Dec
<1	1–2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1–2	3–4	.526	.406	.398	.457	.487	.462	.486	.449	.469	.478	.488	.414		
3	4	.796	.831	.799	.703	.653	.674	.651	.654	.646	.630	.537	.564	.560	.510
4	5	.160	.153	.149	.096	.036	.050	.074	.060	.079	.074	.073	.094	.108	.088
3–4	4–5	.696	.709	.623	.478	.395	.536	.578	.561	.491	.420	.374	.397		
4–5	6–7	.153	.144	.142	.147	.121	.151	.164	.154	.180	.218	.229	.197		
6–7	8–9	.286	.245	.182	.176	.117	.195	.225	.224	.224	.225	.224	.223		
8–9	10–11	.075	.113	.041	.036	.024	.039	.057	.056	.066	.083	.072	.069		
10–11	12–13	.085	.068	.037	.039	.037	.043	.049	.051	.058	.070	.046	.049		
12–13	14–15	.027	.021	.003	.012	.005	.011	.021	.023	.028	.043	.017	.015		
14–15	16–17	.010	.015	0	.013	.011	.011	.015	.014	.021	.037	.004	.003		
16–17	18–19	.007	.011	0	.006	.009	.004	.009	.011	.020	.036	.033	.006		
18–19	20–21	.581	.584	.588	.624	.660	.653	.636	.616	.600	.623	.617	.562		
20–21	22–23	.561	.493	.492	.481	.479	.496	.521	.535	.507	.536	.519	.466		
22–23	24–25	.406	.368	.385	.378	.425	.393	.424	.447	.425	.446	.412	.352		
24–25	26–27	.435	.405	.388	.399	.468	.500	.553	.616	.630	.675	.654	.567		
26–27	28–29	.398	.367	3.92	.416	.485	.547	.643	.732	.731	.771	.773	.671		
38–29	30–31	.813	.820	.820	.809	.855	.877	.915	.959	.978	.994	.992	.986		

Note. — Source: Air Force Systems Command, Air Force Human Resources Laboratory, Computational Sciences Division 25 July 1977.

Using the information in Tables A1 and A2 the probabilities of years of service, P_i, can be obtained and are shown in Table A3. The P_i represent the conditional probabilities that an individual leaves the Air Force in his ith year of service given that he has remained up to that point, that is

$$P_i = S_i \prod_{j=1}^{16} S_j.$$

These calculations used the midpoints of the years of service intervals. Mean time in the Air Force (M) is, then,

$$M = \sum_{i=1}^{16} P_i \bar{Y}_i,$$

where $\bar{Y}_i$ is the midpoint given in Column 1 of Table A3.

The promotion probabilities in Table A4 were developed from cross-sectional data.

Table A3. **Probabilities of Years of Service, Mean Years of Service**

Years of Service Equals	1969		1970		1971		1972		1973		1974		1975	
	June	Dec	June	Dec	June	Dec	June	Dec	June	Dec	June	Dec	June	Dec
1.5	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3.5	.249	.241	.240	.248	.250	.249	.250	.247	.249	.250	.250	.243		
4.5	.101	.123	.141	.135	.123	.134	.125	.136	.133	.127	.120	.140		
6.5	.019	.021	.028	.036	.033	.032	.030	.032	.040	.052	.057	.056		
8.5	.025	.027	.029	.035	.028	.033	.032	.036	.039	.041	.043	.049		
10.5	.006	.011	.006	.007	.006	.006	.008	.008	.011	.014	.013	.014		
12.5	.006	.006	.005	.007	.008	.007	.006	.007	.009	.011	.008	.010		
14.5	.002	.002	.0004	.002	.001	.002	.003	.003	.004	.006	.003	.003		
16.5	.001	.001	0	.002	.002	.002	.002	.002	.003	.005	.001	.001		
18.5	.0005	.001	0	.001	.002	.001	.001	.001	.003	.005	.005	.001		
20.5	.017	.021	.036	.042	.050	.035	.028	.032	.034	.033	.038	.047		
22.5	.007	.009	.015	.017	.019	.013	.011	.013	.014	.013	.015	.021		
24.5	.003	.009	.007	.008	.010	.006	.005	.006	.007	.006	.007	.010		
26.5	.002	.006	.004	.005	.006	.004	.003	.003	.004	.003	.004	.007		
28.5	.001	.003	.003	.003	.003	.002	.001	.001	.001	.001	.001	.003		
30.5	.0004	.001	.001	.001	.001	.0004	.0002	.0001	.00003	.00001	.00001	.0001		
Mean	4.028	4.646	5.003	5.437	5.601	5.026	4.717	4.964	5.271	5.404	5.463	6.067		

Note. — Source: Air Force Systems Command, Air Force Human Resources Laboratory, Computational Sciences Division, 25 July 1977.

Table A4. **Total Enlisted Force: Promotion Probabilities**

Probability that an Individual in Paygrades E1–E3 Will Progress to

UAR Date	E4	E5	E6—E9
June 69	0.919	0.787	0.782
Dec 69	0.890	0.759	0.785
June 70	0.987	0.818	0.866
Dec 70	0.940	0.862	0.890
June 71	0.919	0.877	0.869
Dec 71	0.871	0.823	0.842
June 72	0.983	0.854	0.886
Dec 72	0.926	0.846	0.885
June 73	0.864	0.765	0.801
Dec 73	0.854	0.740	0.804
June 74	0.833	0.746	0.796
Dec 74	0.789	0.709	0.749
June 75	0.767	0.653	0.706
Dec 75	0.743	0.621	0.677
June 76	0.802	0.641	0.725
Dec 76	0.769	0.607	0.669

Note. — Source: Air Force Systems Command, Air Force Human Resources Laboratory, Computational Sciences Division, 25 July 1977.

APPENDIX B: TRANSACTIONS

Figure B1 displays the total number of transactions taking place in fiscal years 1971 through 1976. These data are adjusted for the size of the enlisted force in Figure B2.

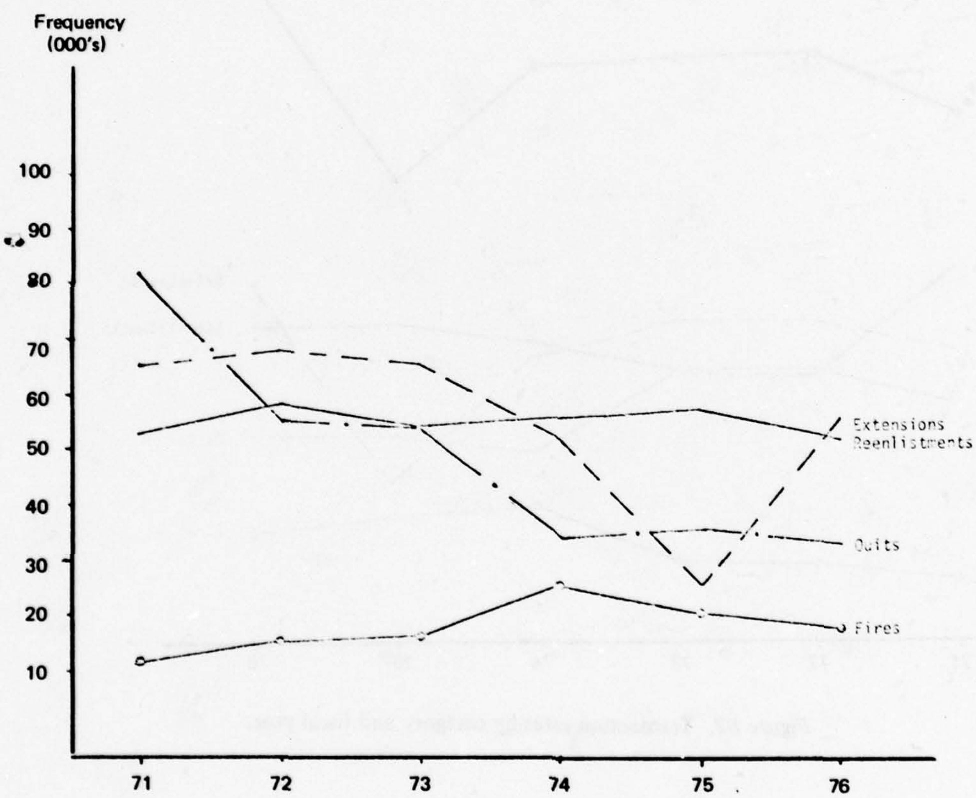

Figure B1. **Gross transactions by category and fiscal year.**

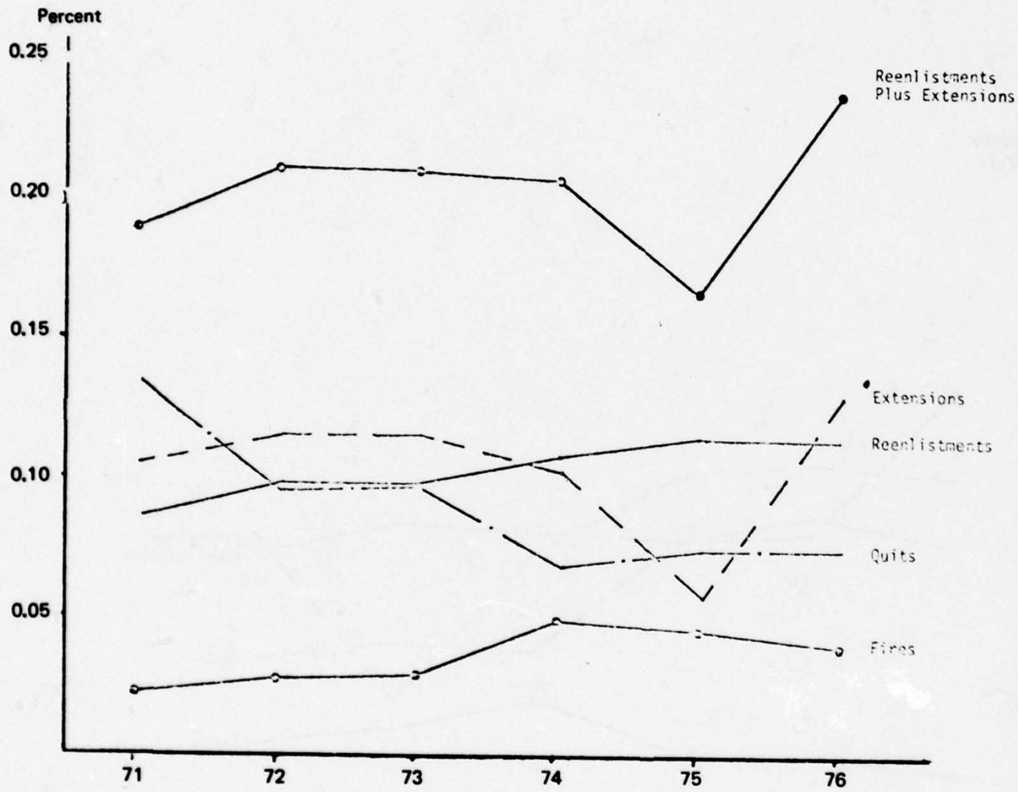

Figure B2. **Transaction rates by category and fiscal year.**

APPENDIX C: QUEUES

This appendix contains figures which display information regarding the queue of enlistees: that is, those individuals who have enlisted but are waiting to report for active duty.

Figure C1 shows, for each month, the number of individuals who are waiting for between 1 and 6 months, inclusive. In Figure C2, the queue length is adjusted for force levels. Mean waiting time in months is given in Figure C3.

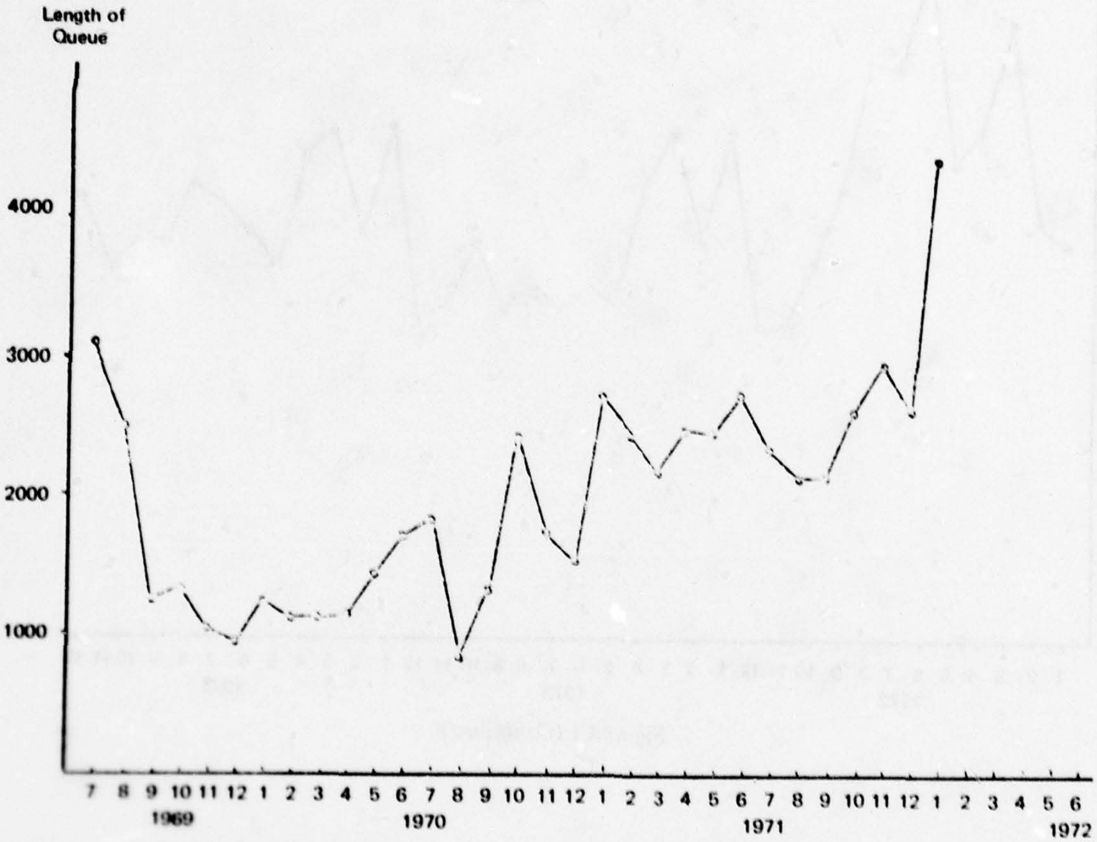

Figure C1. **Number waiting to commence active duty in given month (1–6 months).**

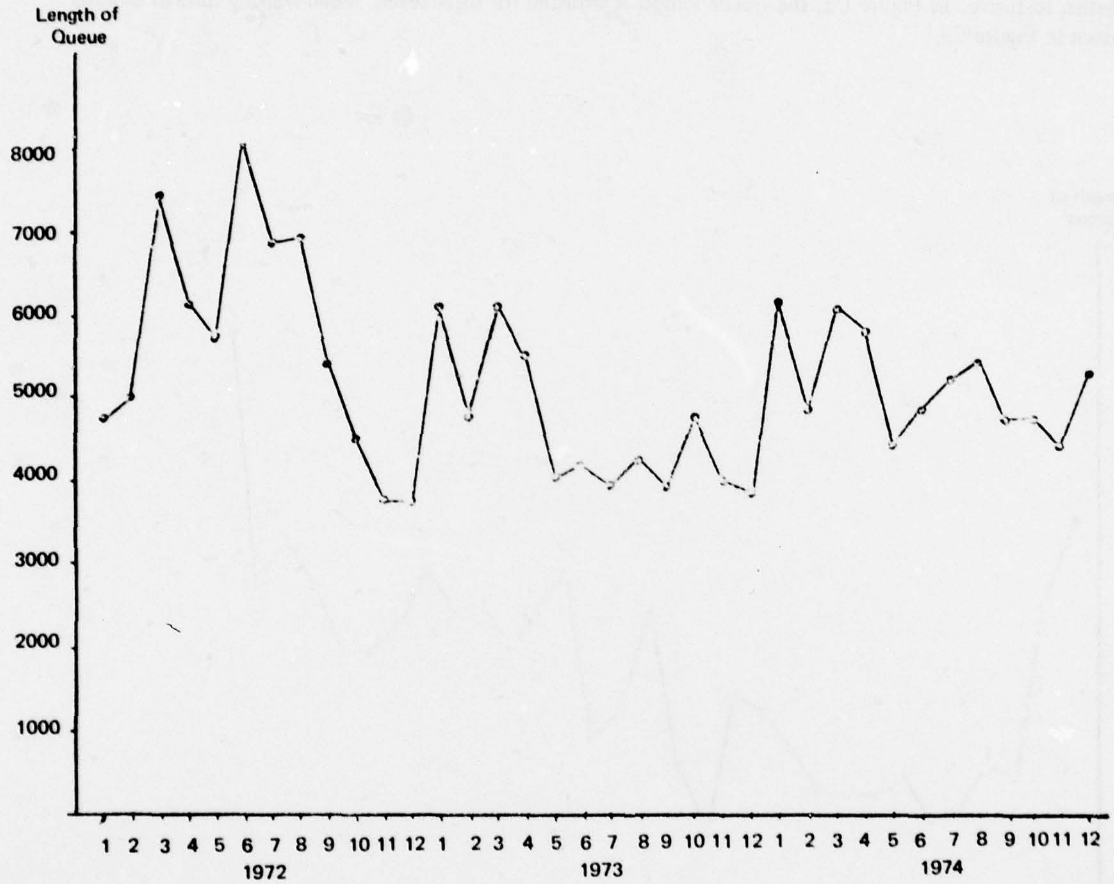

Figure C1 (*Continued*)

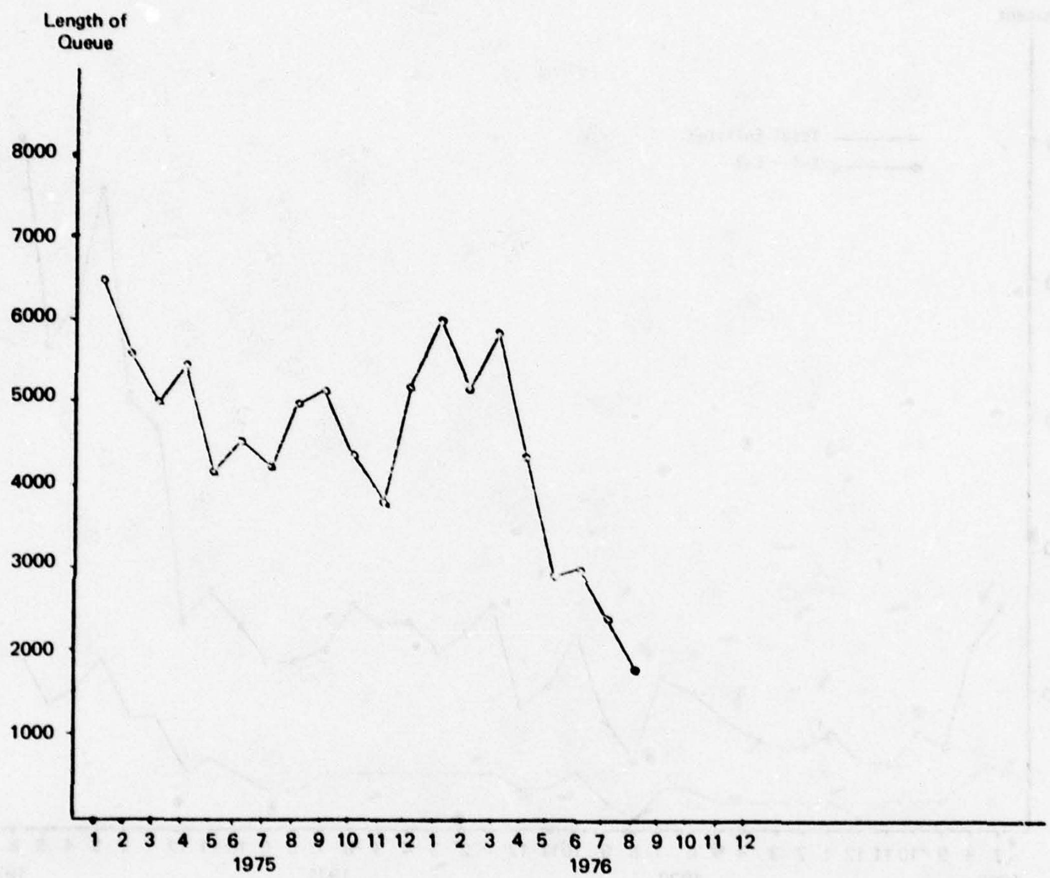

Figure C1 (*Continued*)

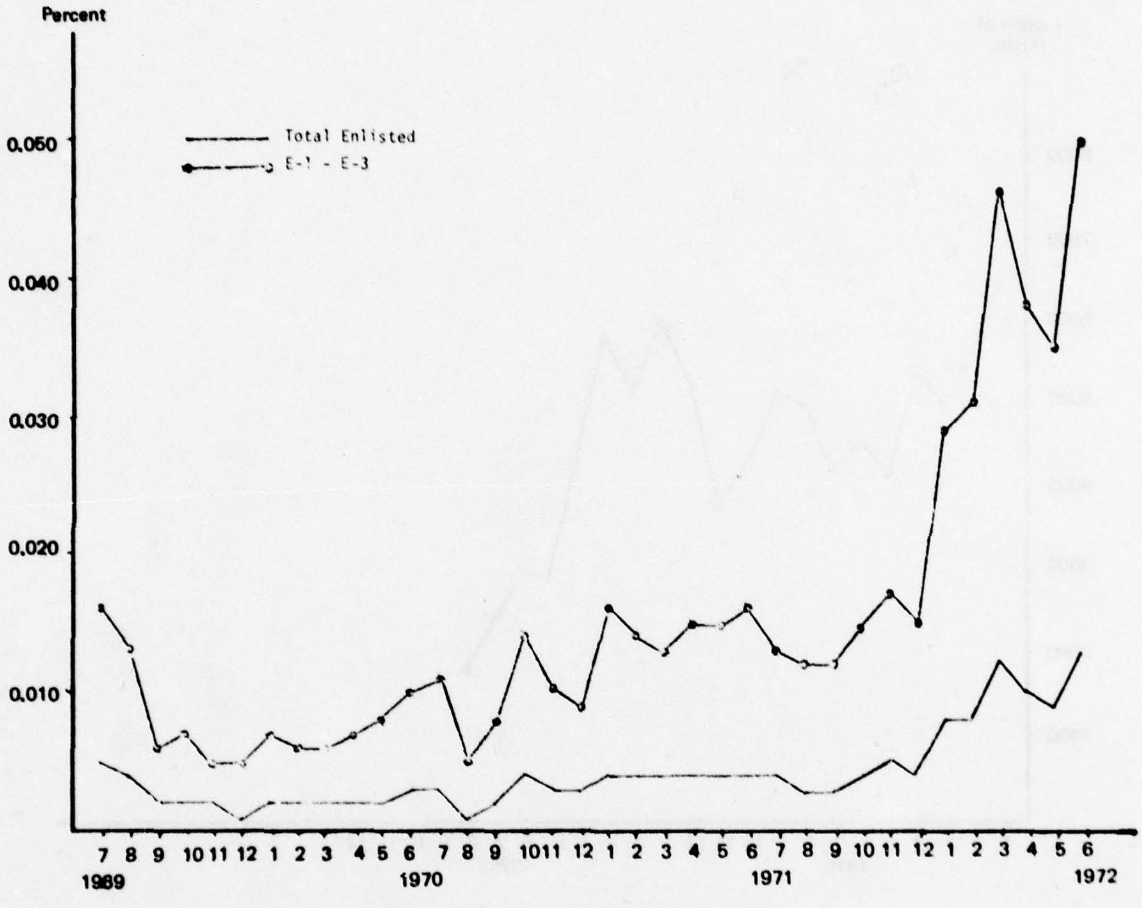

Figure C2. **Length of queue relative to force levels.**

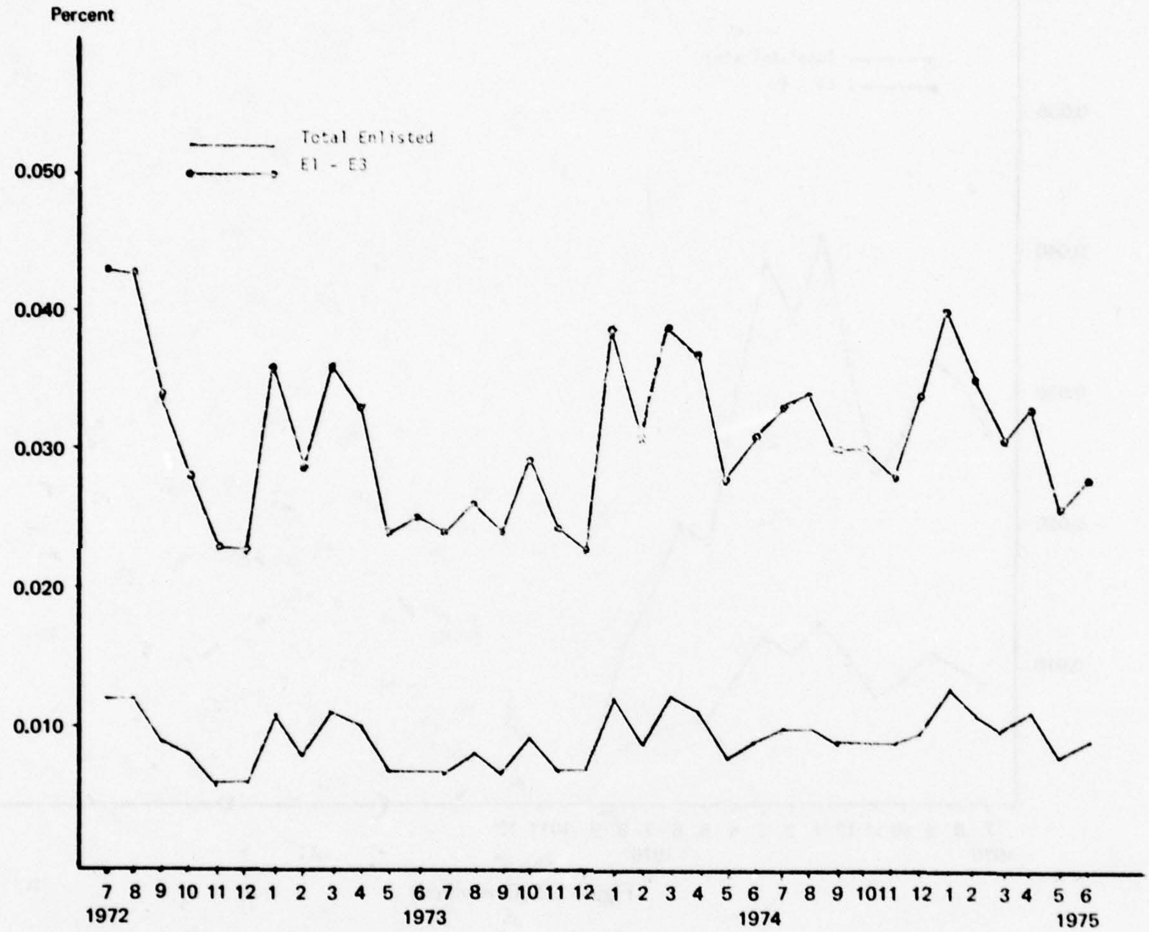

Figure C2 (*Continued*)

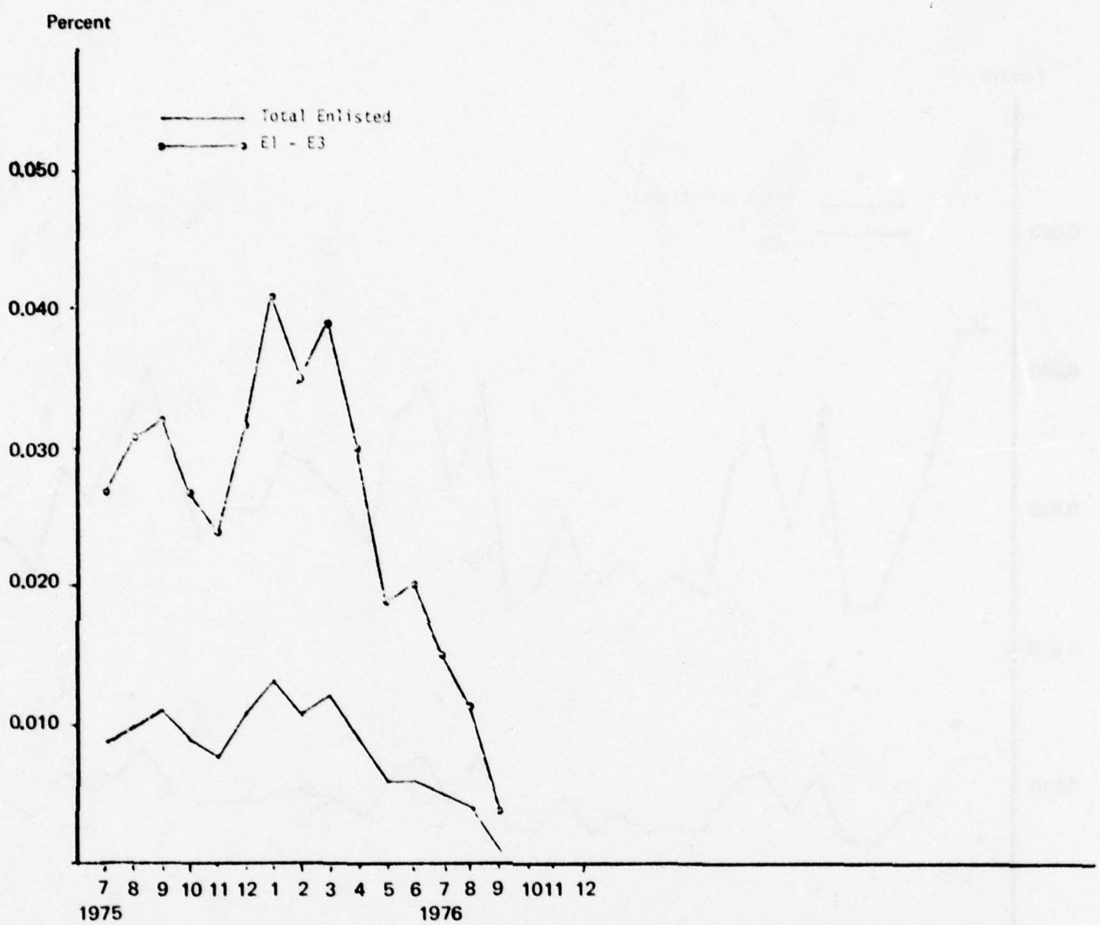

Figure C2 (*Continued*)

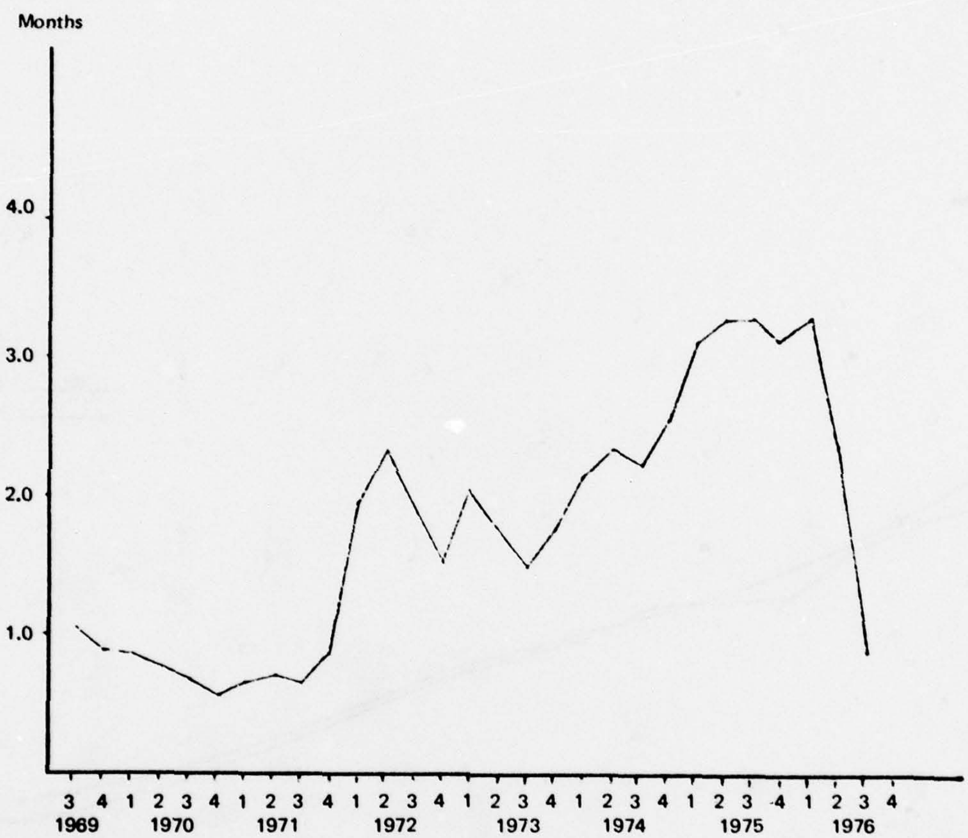

Figure C3. **Mean wait per quarter.**

APPENDIX D: INVENTORIES

Figures D1 through D5 compare actual inventories of personnel in selected paygrades with Congressionally-authorized end-strengths. The "top-six ratios" in Figure D5 refer to the number of individuals in paygrades E-4 through E-9 divided by the number in paygrades E-1 through E-3.

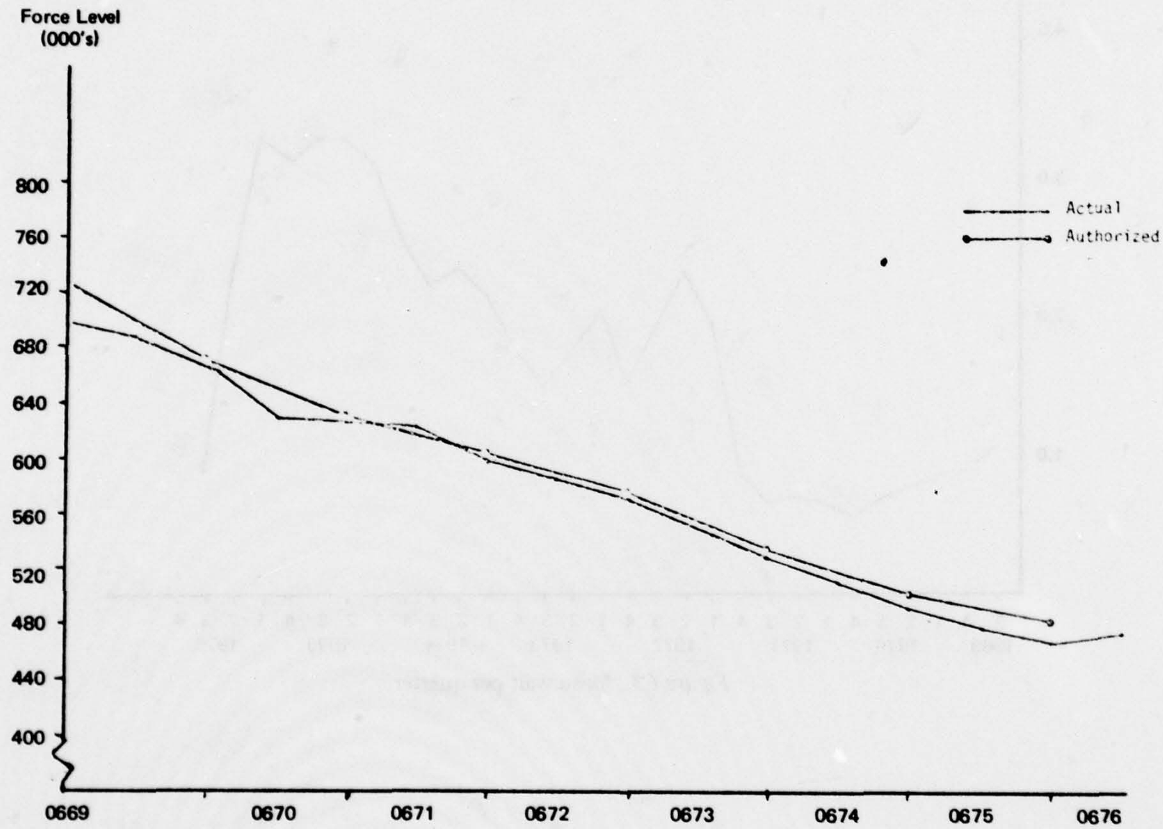

Figure D1. **Total enlisted force: Force levels and authorized end-strength, FY60–FY76.**

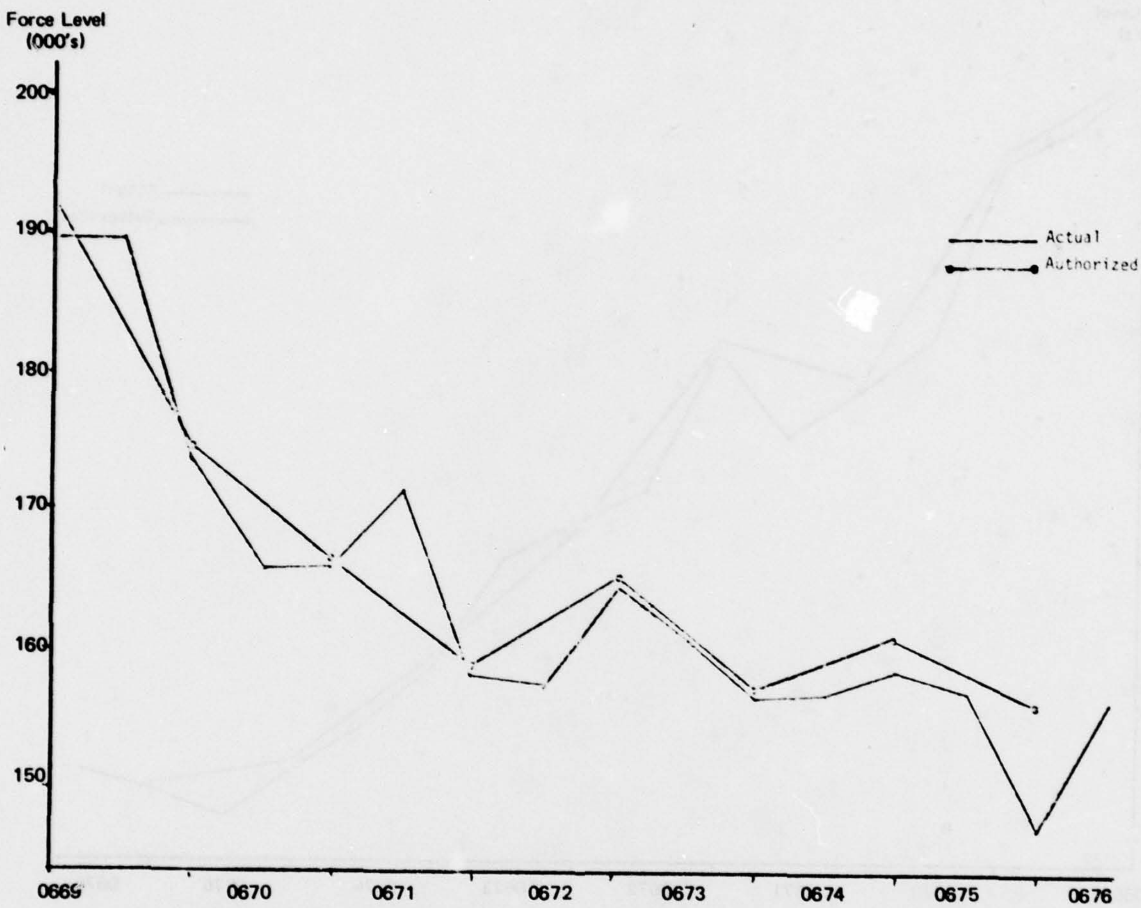

Figure D2. **Paygrades E1–E3: Force levels and authorized end-strength, FY69–FY76.**

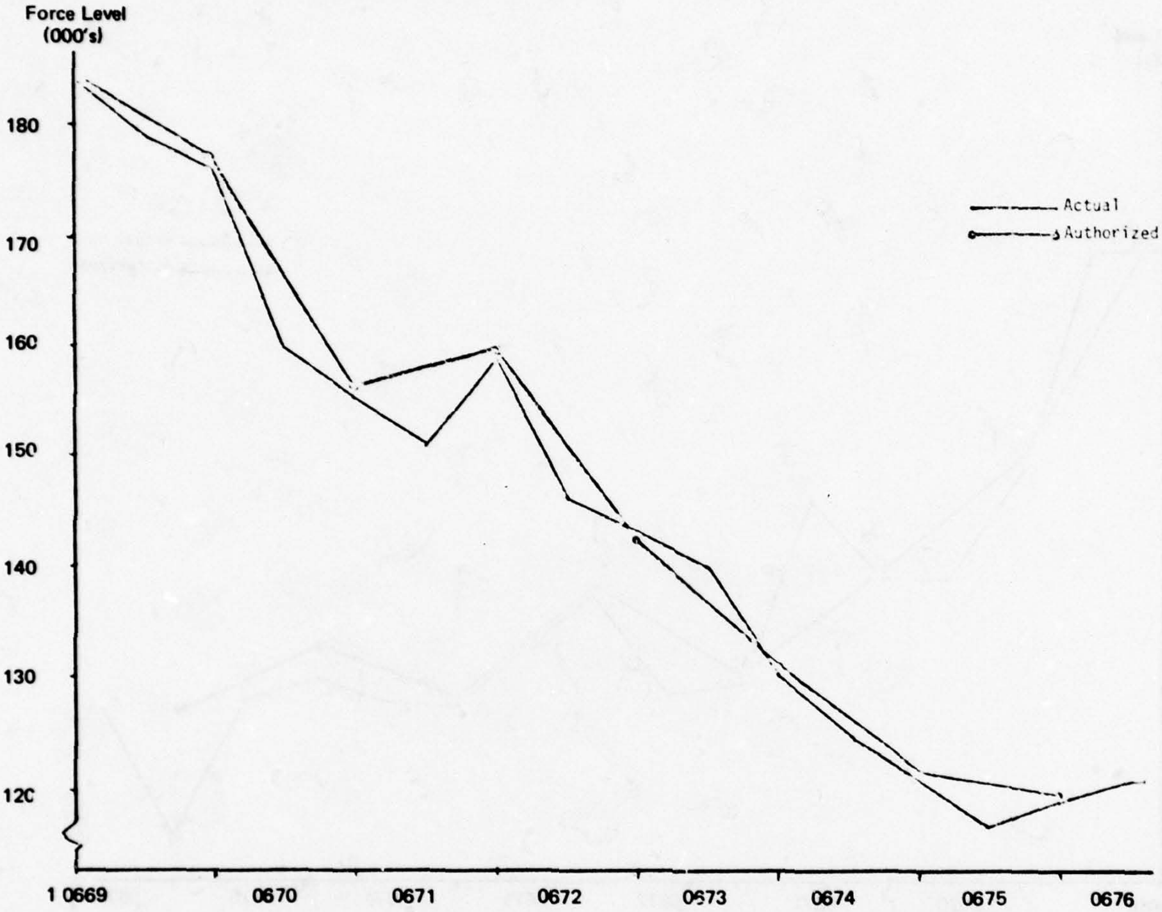

Figure D3. **Paygrade E4: Force levels and authorized end-strength, FY69–FY76.**

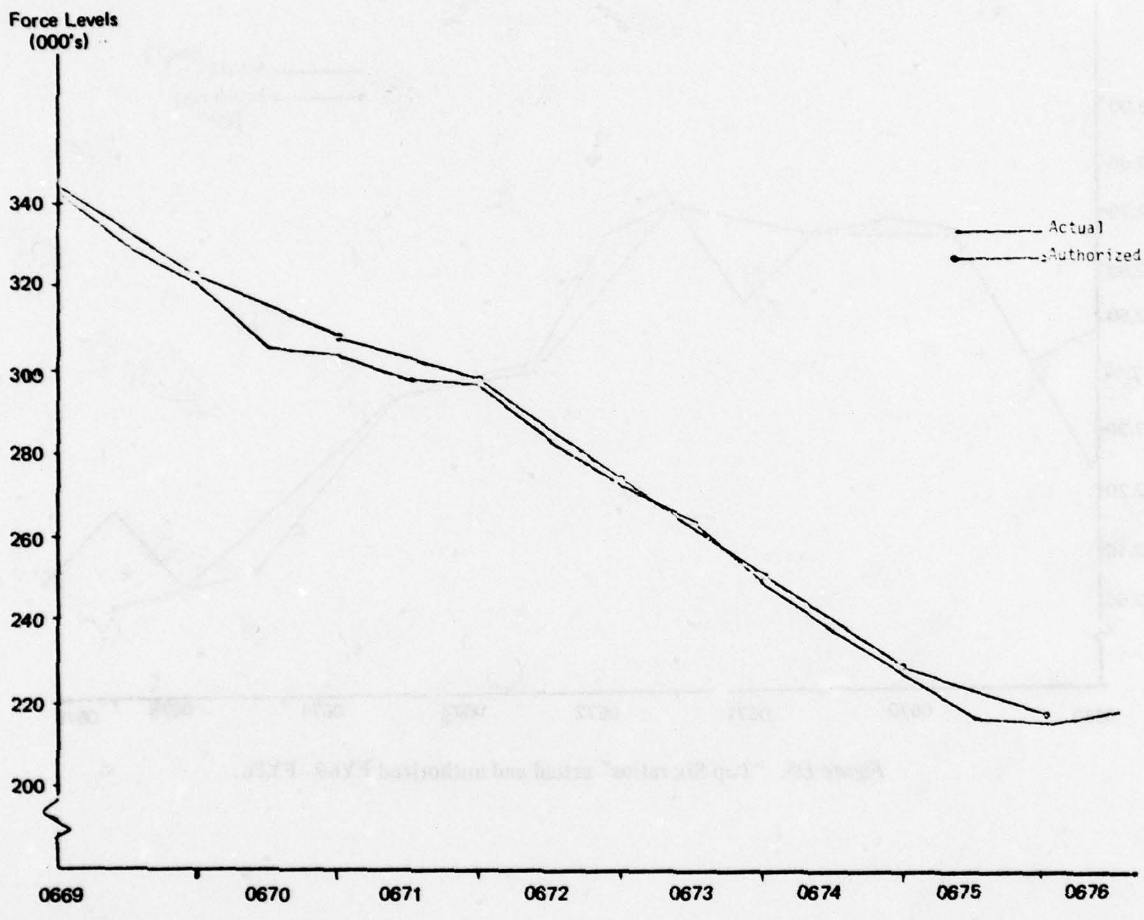

Figure D4. **Paygrades E4–E5: Force levels and authorized end-strength, FY69–FY76.**

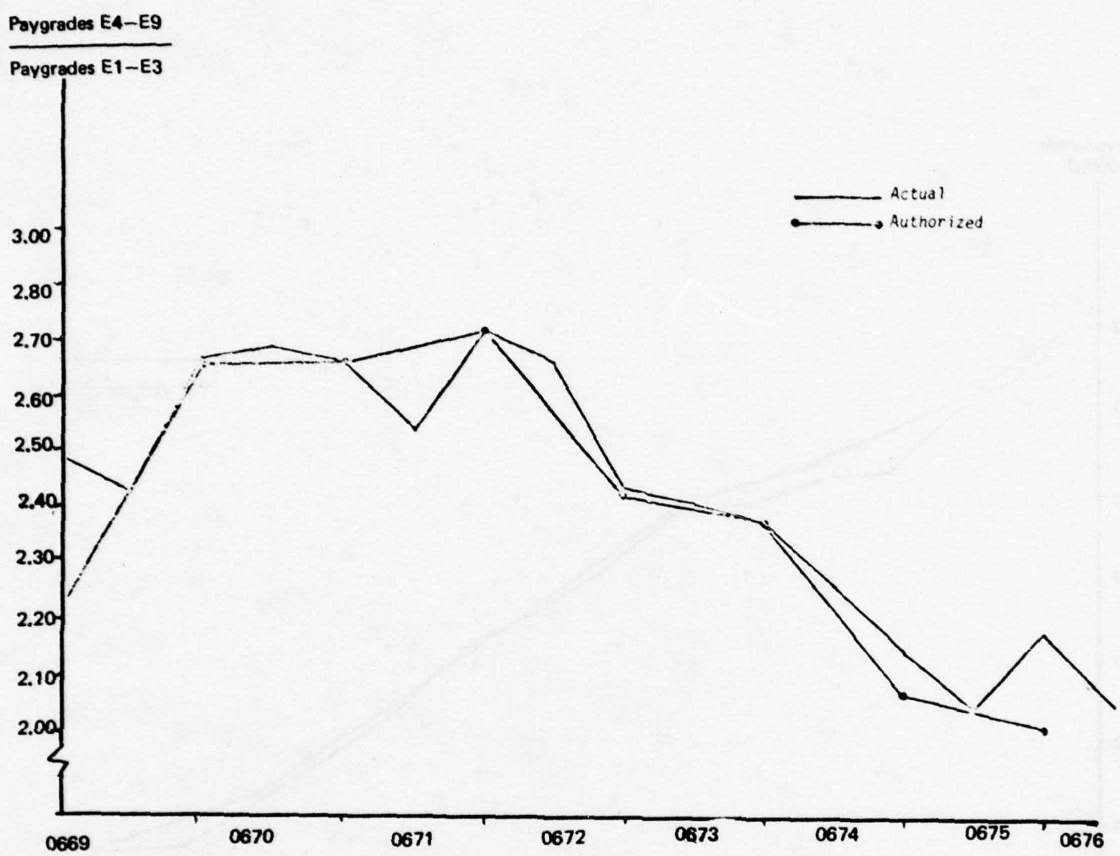

Figure D5. **"Top-Six ratios" actual and authorized FY69–FY76.**

APPENDIX E: AFQT DISTRIBUTIONS

This appendix contains histograms which display the distribution of AFQT scores for selected dates and year of service intervals. The AFQT categories are based on the following score grouping:

AFQT Category	Raw Score
1	93 and above
2	65 – 92
3	31 – 64
4	21 – 30
5	16 – 20
6	10 – 15
7	9 and below

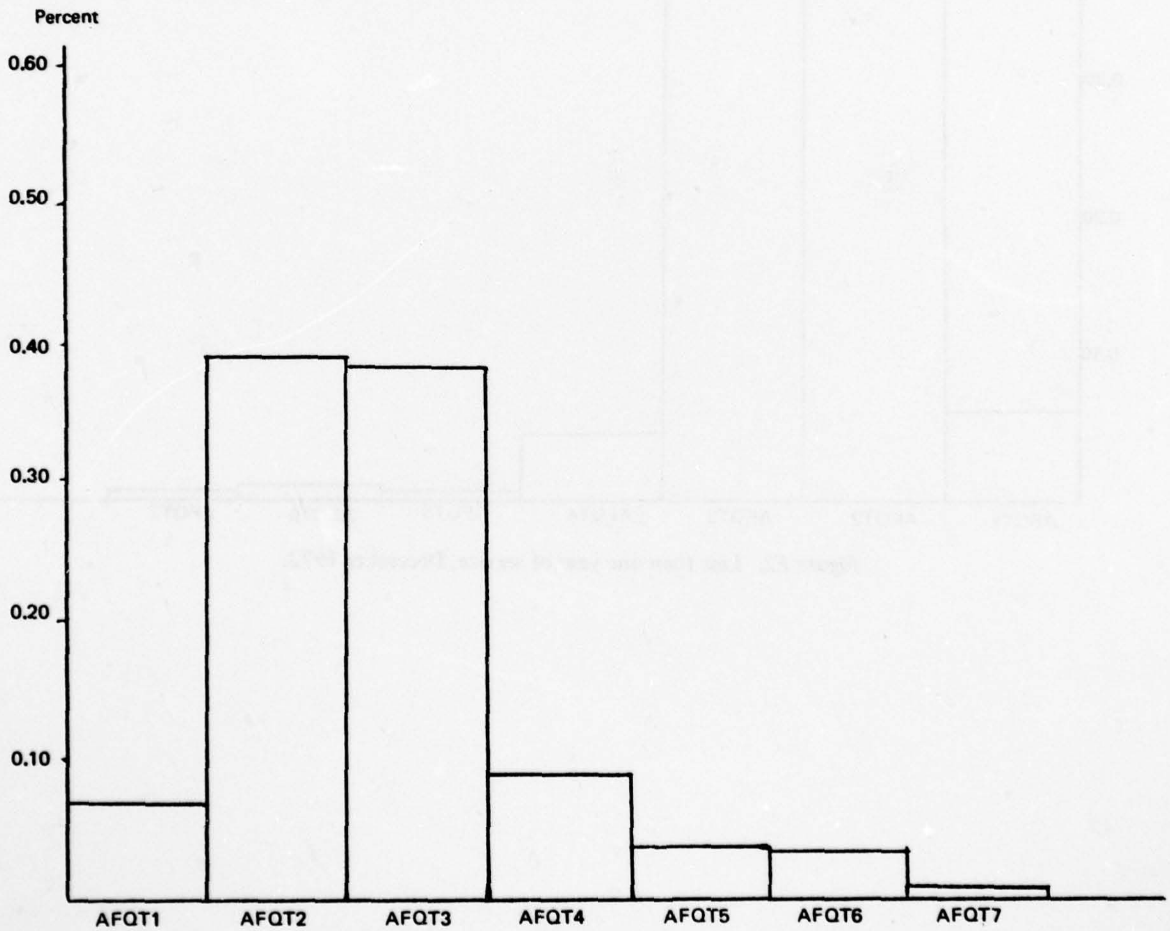

Figure E1. **Less than one year of service , December 1970.**

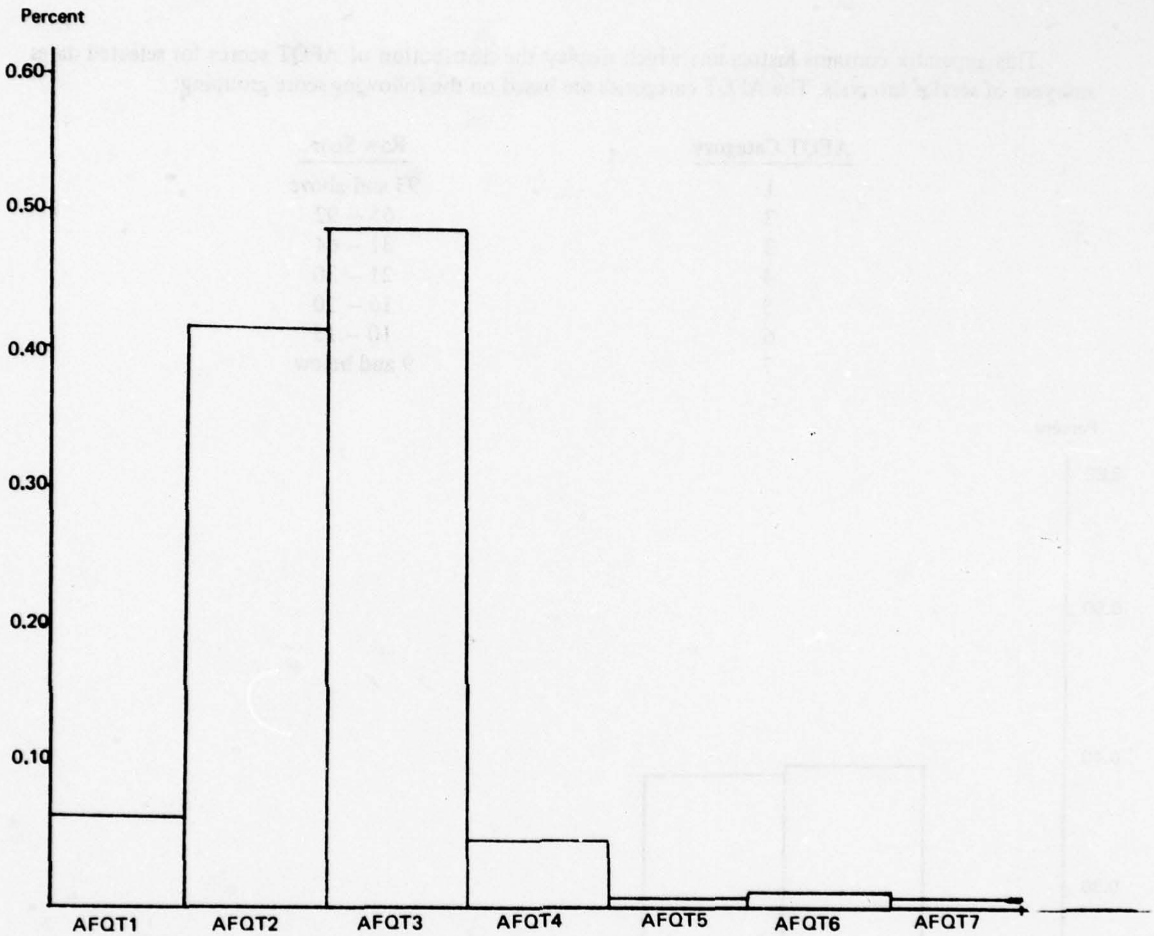

Figure E2. **Less than one year of service, December 1972.**

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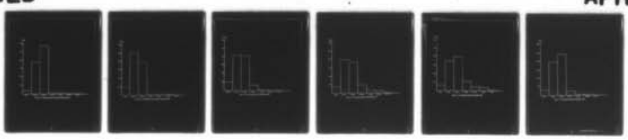

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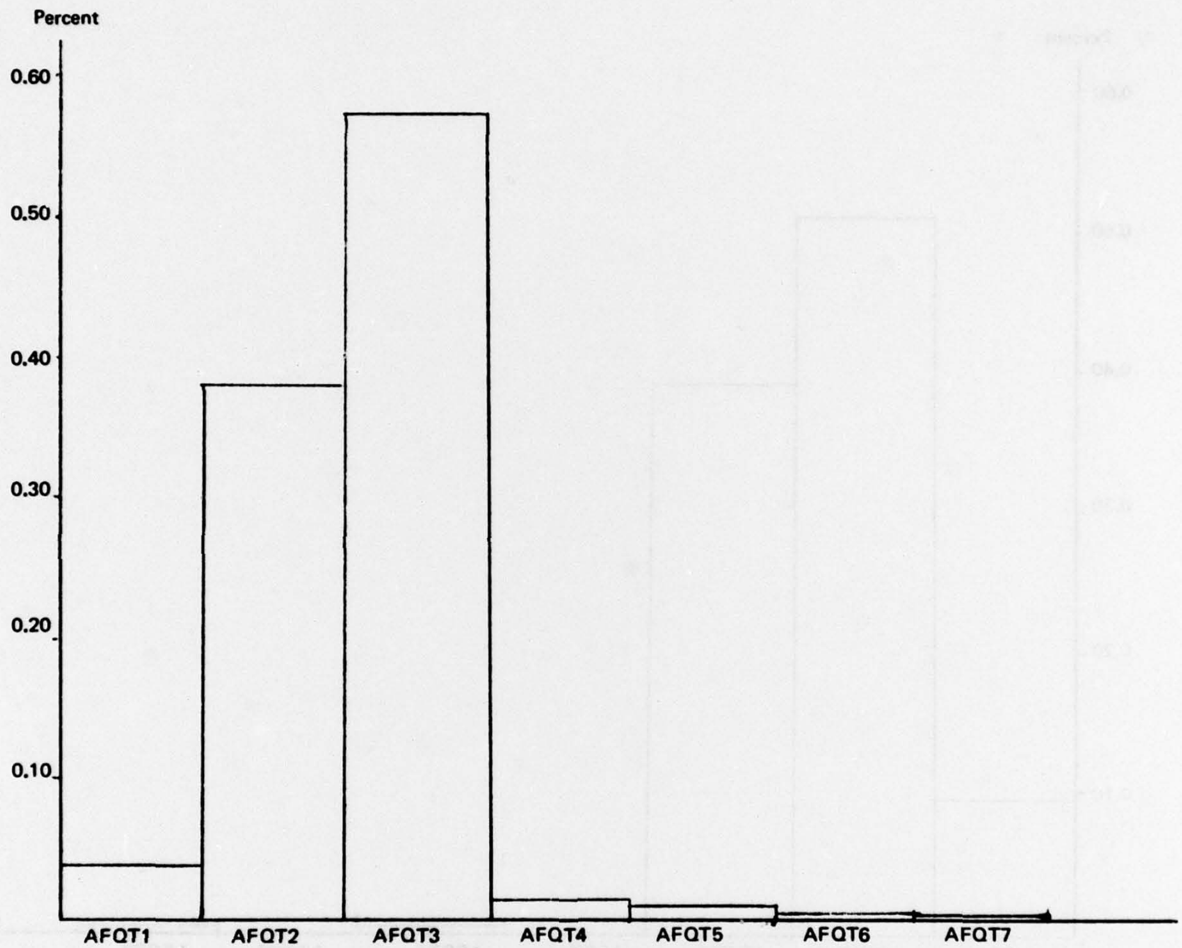

Figure E3. **Less than one year of service, December 1974.**

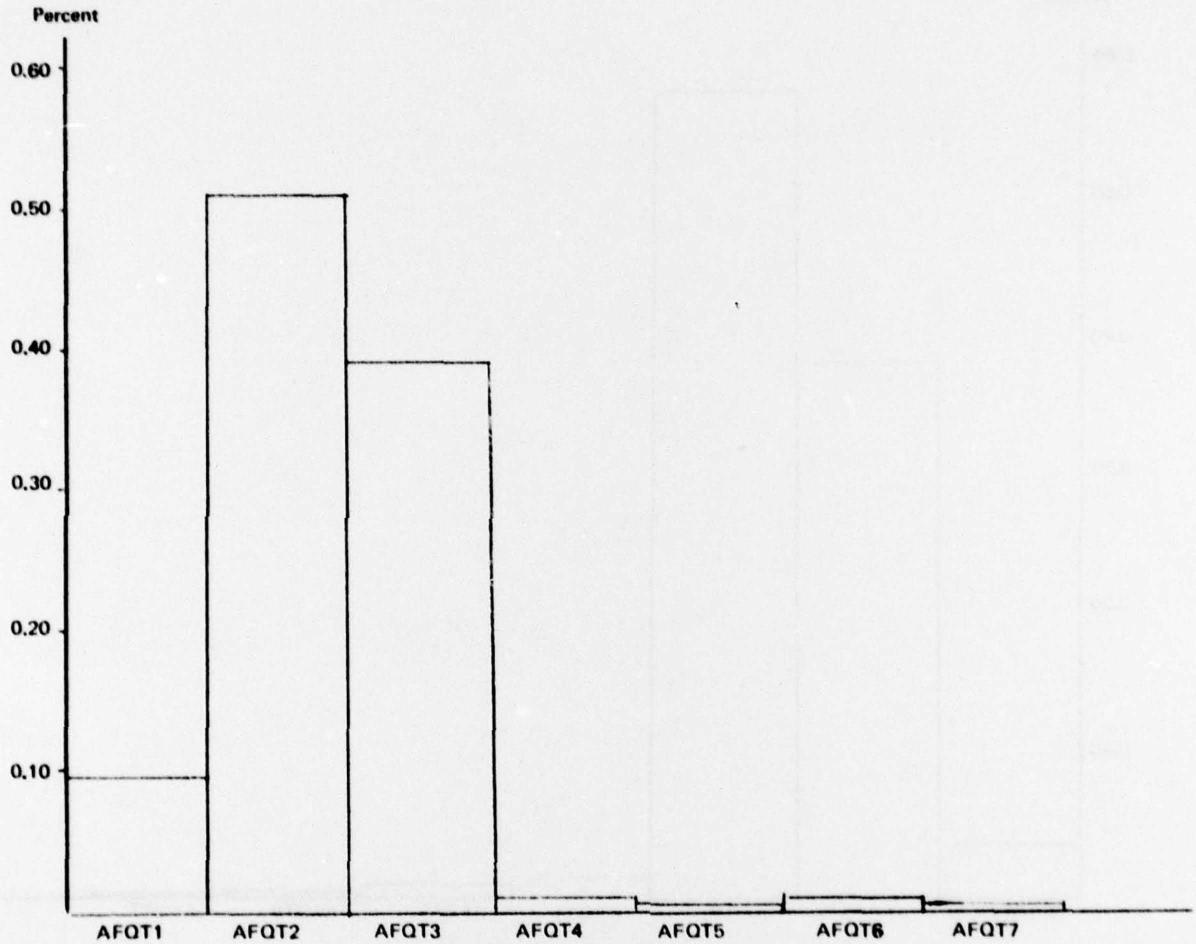

Figure E4. **Less than one year of service, December 1976.**

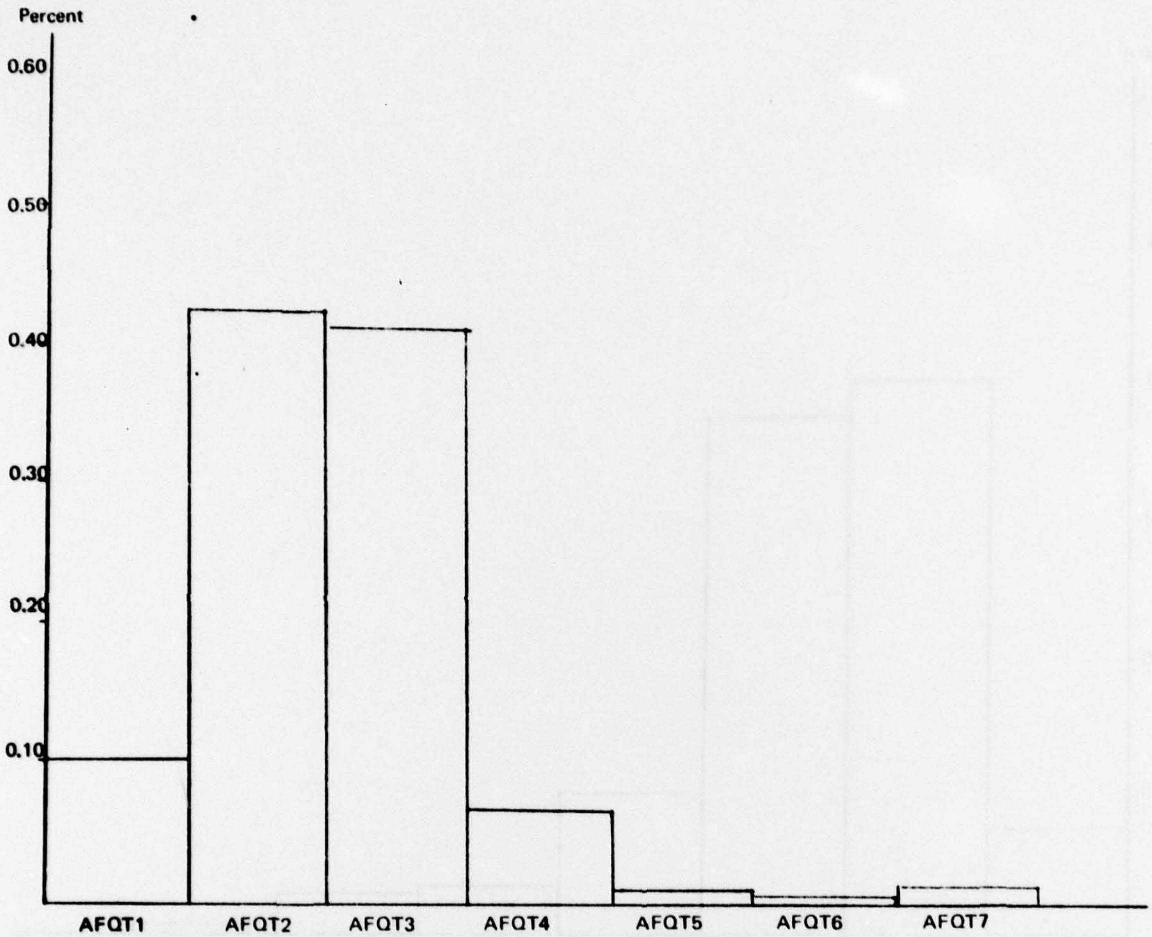

Figure E5. **Four years of service, December 1970.**

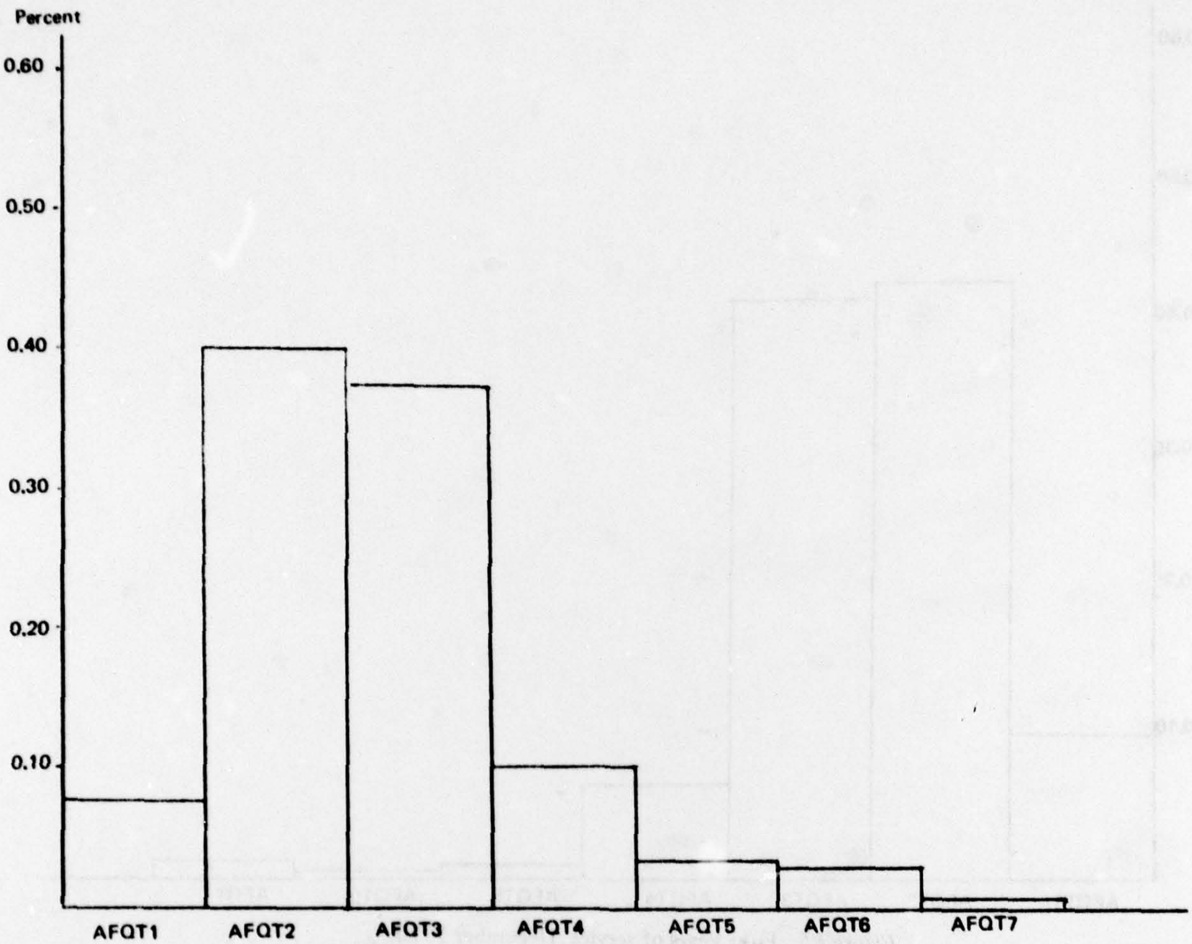

Figure E6. **Four years of service, December 1972.**

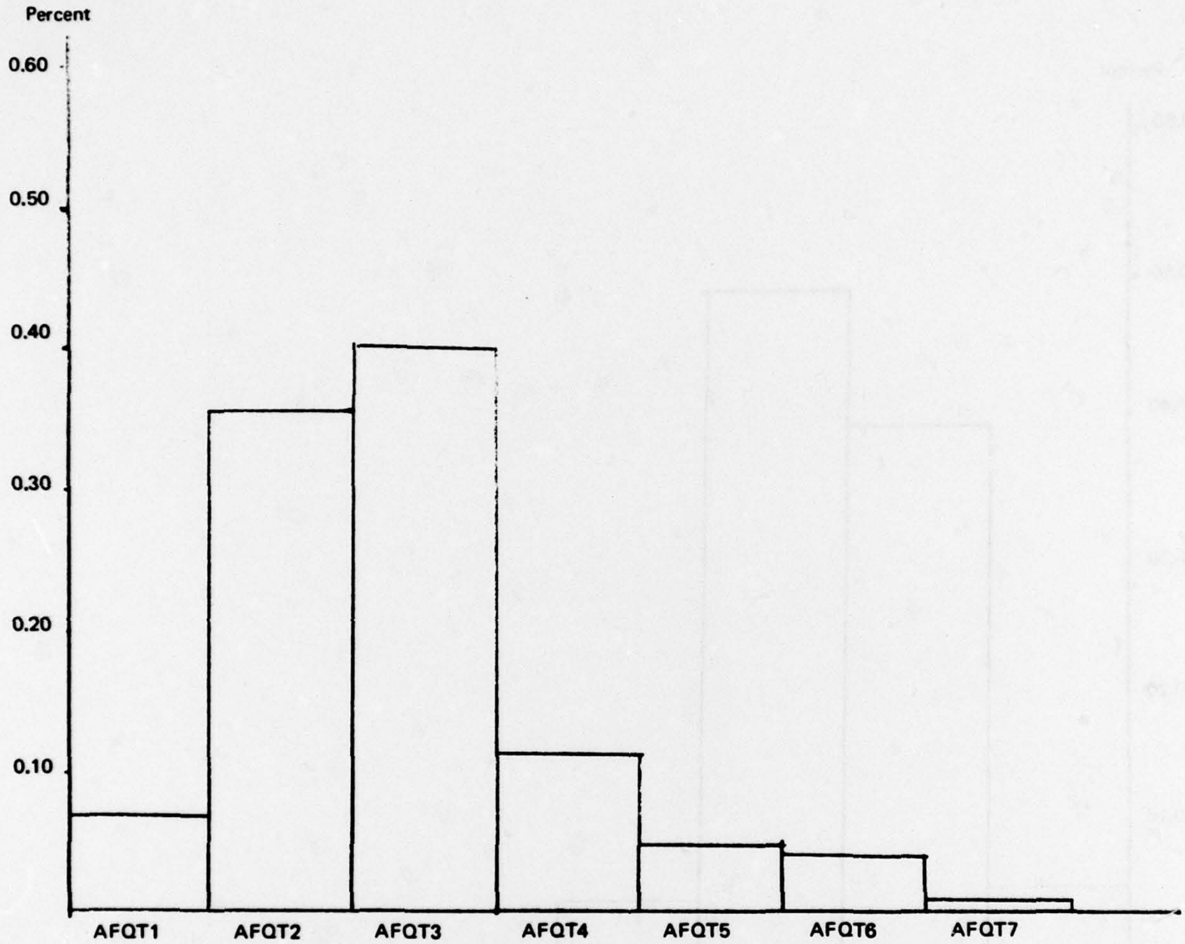

Figure E7. **Four years of service, December 1974.**

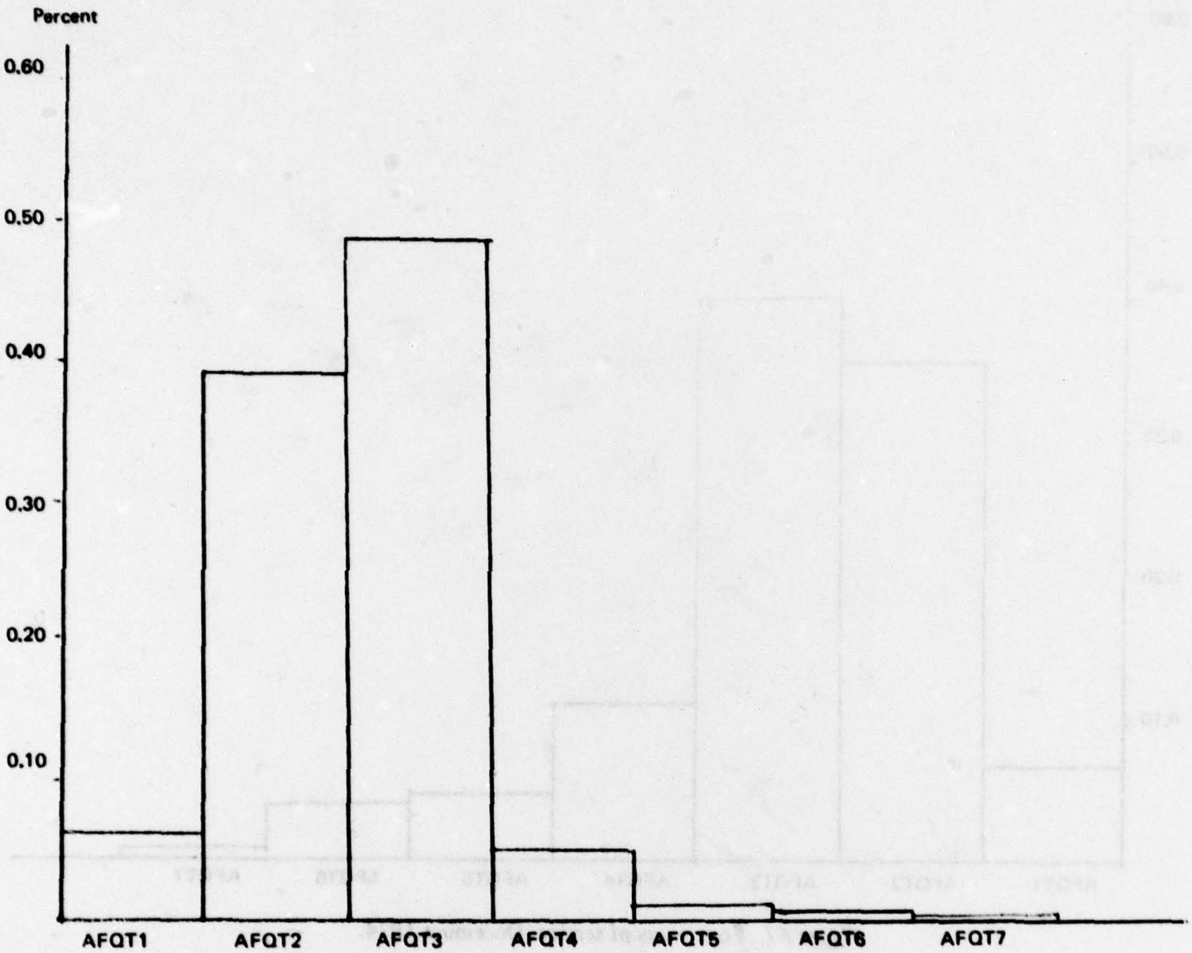

Figure E8. **Four years of service, December 1976.**

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SUPPLY RATE AND EQUILIBRIUM INVENTORY OF AIR FORCE ENLISTED PER--ETC(U)
MAY 78 A S DEVANY, T R SAVING, W F SHUGHART F33615-77-C-0056
UNCLASSIFIED AFHRL-TR-78-10 NL

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Errata

Number	First Author	Title
AFHRL-TR-76-87 (AD-A037 522)	Jensen	Armed Services Vocational Aptitude Battery Development (ASVAB Forms 5, 6, and 7)
AFHRL-TR-77-28 (AD-A044 525)	Hunter	Validation of a Psychomotor/Perceptual Test Battery
AFHRL-TR-77-53 (AD-A048 120)	Mathews	Screening Test Battery for Dental Laboratory Specialist Course: Development and Validation
AFHRL-TR-77-74 (AD-A051 962)	Mathews	Analysis Aptitude Test for Selection of Airmen for the Radio Communications Analysis Specialist Course: Development and Validation
AD-A058 097 AFHRL-TR-78-10 (AD-A058 097)	DeVany	Supply Rate and Equilibrium Inventory of Air Force Enlisted Personnel: A Simultaneous Model of the Accession and Retention Markets Incorporating Force Level Constraints
AFHRL-TR-78-74 (AD-A066 659)	Leisey	Characteristics of Air Force Accessions: January 1975 to June 1977
AFHRL-TR-78-82 (AD-A063 656)	Mathews	Prediction of Reading Grade Levels of Service Applicants from Armed Services Vocational Aptitude Battery (ASVAB)
AFHRL-TR-79-29 (AD-A078 427)	Hendrix	Pre-Enlistment Person-Job Match System
AFHRL-TR-79-83 (AD-A090 499)	Gustafson	Recursive Forecasting System for Person-Job Match

Due to norming problems encountered with ASVAB Forms 5, 6, and 7, percentile scores derived from these test forms are in error. While the relative ranking of individuals by their percentile scores would not be affected by the norming errors, their absolute score values would be different. Therefore, descriptive statistics reported in the subject technical reports above are erroneous; other types of analyses in the report which use ASVAB percentile scores should be interpreted with caution.

NANCY GUINN, Technical Director
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